

Seminar 8. Linear regressions

Task 1. Linearization

Check if such functions can be transformed to linear regression form. Provide necessary calculations, which prove or reject your guess.

1. $y = \frac{1}{\alpha + \beta \cdot e^x};$

2. $y = \frac{e^x}{\alpha + \beta \cdot e^x};$

3. $y = 1 + \alpha(1 - x^\beta);$

4. $y^2 = \alpha e^{\beta x}.$

5. $y_t = \alpha \cdot e^{\beta x_t} \cdot \varepsilon_t;$

6. $y_t = \alpha \cdot \ln x_1 + \beta \ln \gamma^{x_2} + \delta e^{x_3} + \varepsilon_t;$

7. $y_t = \alpha e^{-\beta x_t} + \varepsilon_t;$

8. $y_t = e^{\alpha + \beta x_t + \varepsilon_t};$

9. $y_t = \frac{\alpha}{\beta - x_t} + \varepsilon_t;$

10. $y_t = \alpha \cdot \gamma \cdot x_1 \cdot x_2^2 \cdot x_3 + \beta \ln \gamma^{x_1 x_2} + \delta e^{x_3} + \varepsilon_t.$

Task 2. Best choice

Using data in the table estimate the following regressions.

- $y_t = \alpha_0 + \alpha_1 x_{1t} + \varepsilon_t;$
- $y_t = \beta_0 + \beta_1 x_{2t} + \varepsilon_t;$
- $y_t = \gamma_0 + \gamma_1 x_{1t} + \gamma_2 x_{2t} + \varepsilon_t.$

Choose the best one based on econometric tests.

y	x_1	x_2
28,4	635,7	92,9
32,0	688,1	94,5
37,7	753,0	97,2
40,6	796,3	100,0
47,7	868,5	104,2
52,9	935,5	109,8
58,5	982,4	116,3
64,0	1063,4	121,3

y	x_1	x_2
75,9	1171,1	125,3
94,4	1306,6	133,1
131,9	1412,9	147,7
126,9	1528,8	161,2
155,4	1702,2	170,5
185,8	1899,5	181,5
217,5	2127,6	195,4
260,9	2368,5	217,4

Task 3. Enterprise output

Table consists statistics regarding 15 enterprises with similar production.

№	Output, mln. UAH, y	Average labour productivity, UAH/hour, x_1	The effectiveness of capital assets, UAH per 1000 UAH., x_2
1	26	37	39
2	33	33	40
3	24	15	35
4	29	36	48
5	42	26	53
6	24	24	42
7	52	15	54
8	56	33	54
9	26	44	50
10	45	34	53
11	27	63	46
12	54	8	50
13	34	44	43
14	48	43	55
15	45	31	51

1. Estimate regression coefficients: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$.
2. Check the model for significance, $\alpha = 0,1$.
3. Forecast output if the average labour productivity reaches $x_1 = 45$ and the effectiveness of capital assets is $x_2 = 59$.
4. Define elasticity coefficients and chose the most important factor.

Task 4. Advertising

Firm's output can be written in the form: $y_t = \beta_0 + \beta_1 p_t + \beta_2 a_t + \beta_3 a_t^2 + \varepsilon_t$, where y_t – firm's output, p_t – price for one unit, a_t – advertising expenses. Cost price is 1.8.

1. Using data in the table estimate the model and test it for significance.
2. Calculate the optimal price if advertising expenses are 6.1.
3. Calculate optimal expenses for advertising, if competitive price is 4.9.

t	y_t	p_t	a_t
1	508	3,78	3,59
2	550	4,36	2,41
3	379	4,40	4,29
4	709	3,97	1,58
5	248	4,48	4,54
6	598	3,01	0,14

t	y_t	p_t	a_t
7	353	2,88	4,61
8	772	2,88	2,19
9	496	4,63	2,54
10	644	3,43	2,39
11	390	4,53	3,99
12	591	4,78	2,07

t	y_t	p_t	a_t
13	382	4,83	3,49
14	614	4,45	2,59
15	528	4,36	3,09
16	495	4,72	1,51
17	828	2,95	1,01
18	554	3,94	1,89

t	y_t	p_t	a_t
19	522	4,22	3,45

t	y_t	p_t	a_t
20	603	4,08	0,77

Task 5. Elasticity case

The manager thinks that his business income may depend on the prices of products, cost of capital equipment, labour force, exchange rate, capital investment in the sector (all parameters are presented in the table). Based on dynamics of parameters help manager to specify the best econometric model. Define the most important factors.

Quarter	Income, mln UAH, y	Price of output, UAH, p	Capital equipment, mln. UAH, K	Labor force, th. people*hours, L	Exchange rate UAH/USD, e	Investments, mln. UAH, I
1997/Q1	0,751	2,10	103	204	185,74	112
1997/Q2	0,676	3,19	107	204	184,95	211
1997/Q3	2,204	3,71	108	207	185,79	401
1997/Q4	2,216	3,79	116	208	188,19	691
1998/Q1	2,433	4,04	117	214	196,66	209
1998/Q2	2,482	4,15	133	215	204,94	321
1998/Q3	2,502	4,23	130	217	235,69	345
1998/Q4	3,182	4,94	121	217	342,52	799
1999/Q1	3,472	4,97	144	217	355,69	223
1999/Q2	3,955	5,82	144	218	393,56	361
1999/Q3	4,143	6,59	145	218	430,69	483
1999/Q4	4,421	6,66	148	221	472,23	1039
2000/Q1	4,432	6,88	149	222	546,41	319
2000/Q2	5,054	7,57	153	224	541,35	482
2000/Q3	5,186	7,59	154	224	543,90	609
2000/Q4	5,276	7,77	160	226	543,87	1425
2001/Q1	5,368	7,93	162	227	543,14	473
2001/Q2	5,415	7,94	166	228	540,75	727
2001/Q3	5,954	9,01	167	229	535,23	899
2001/Q4	5,977	9,01	176	230	529,73	1109
2002/Q1	6,791	9,11	178	231	531,86	577
2002/Q2	7,042	9,18	183	232	532,82	872
2002/Q3	8,305	9,43	203	204	532,91	932
2002/Q4	8,565	9,53	221	204	533,10	2081

Task 6. Ukrainian GDP case

Based on Ukrainian GDP for 1993-2003, build linear trend model $y_t = \beta_0 + \beta_1 t + \varepsilon_t$.

Quarter	GDP, mln. USD
1993/Q1	53
1993/Q2	128
1993/Q3	470
1993/Q4	831
1994/Q1	1478
1994/Q2	1982
1994/Q3	2979
1994/Q4	5597
1995/Q1	8318
1995/Q2	10694
1995/Q3	16102
1995/Q4	19402
1996/Q1	16688
1996/Q2	17867
1996/Q3	22510
1996/Q4	24454
1997/Q1	18728
1997/Q2	20485
1997/Q3	26076
1997/Q4	28076
1998/Q1	20871
1998/Q2	23367

Quarter	GDP, mln. USD
1998/Q3	28908
1998/Q4	29447
1999/Q1	24980
1999/Q2	29196
1999/Q3	37633
1999/Q4	35317
2000/Q1	32309
2000/Q2	37889
2000/Q3	51238
2000/Q4	48634
2001/Q1	39201
2001/Q2	46481
2001/Q3	58999
2001/Q4	59509
2002/Q1	43699
2002/Q2	49893
2002/Q3	64081
2002/Q4	63259
2003/Q1	51206
2003/Q2	59937
2003/Q3	65413

1. Test model for significance, $\alpha = 0.05$.
2. Using data for 2002, calculate the forecast on three quarters of 2003, compare them with real data, and define RMSPE.

Task 7. Linear model

For the model $y_t = \beta_0 + \beta_1 x_{1t} + \varepsilon_t$ calculate coefficient estimates $\hat{\beta}_0, \hat{\beta}_1$.