

Seminar 6. Hypothesis testing

Examples with answers

1. The Food and Nutrition Board of the National Academy of Sciences states that the recommended daily allowance (RDA) of iron for adult females under the age of 51 is 18 milligrams (mg). A sample of iron intake in was obtained during a 24-hour period from 45 randomly selected adult females under the age of 51. It revealed that the sample mean was 14.68 mg. At the 1 percent significance level, does the data suggest that adult females under the age of 51 are, on average, getting less than the RDA of 18 mg of iron? Assume that the population standard deviation is 4.2 mg.

Answer: At the 1 percent significance level, the data provides sufficient evidence to conclude that adult females under the age of 51 are, on average, getting less than the RDA of 18 mg of iron.

2. Is there a significant (or only random) difference in the average cycle time to deliver a pizza from Pizza Company A vs. Pizza Company B. This is the data collected from a sample of deliveries of Company A and Company B.

Pizza Company A Versus Pizza Company B Sample Deliveries	
A	B
20.4	20.2
24.2	16.9
15.4	18.5
21.4	17.3
20.2	20.5
18.5	
21.5	

Answer: Since the p-value is 0.289, i.e. greater than 0.05 (or 5 percent), it can be concluded that there is no difference between the means.

3. Duracell manufactures batteries that the CEO claims will last an average of 300 hours under normal use. A researcher randomly selected 20 batteries from the production line and tested these batteries. The tested batteries had a mean life span of 270 hours with a standard deviation of 50 hours. Do we have enough evidence to suggest that the claim of an average lifetime of 300 hours is false?

Answer: Reject the Null Hypothesis ($t=-2.68$)

4. The television habits of 30 children were observed. The sample mean was found to be 48.2 hours per week, with a standard deviation of 12.4 hours per week. Test the claim that the standard deviation was at least 16 hours per week.

Answer: The variation in television watching was less than 16 hours per week ($\chi^2=17.418$).

5. Samples from two makers of ball bearings are collected, and their diameters (in inches) are measured, with the following results:

Acme: $n_1=80$, $s_1=0.0395$,

Bigelow: $n_2=120$, $s_2=0.0428$.

Assuming that the diameters of the bearings from both companies are normally distributed, test the claim that there is no difference in the variation of the diameters between the two companies.

Answer: There is insufficient evidence to conclude that the diameters of the ball bearings in the two companies have different standard deviations ($F=0.8517$).

Problems

1. An engineer measured the Brinell hardness of 25 pieces of ductile iron that were subcritically annealed. The resulting data were:

170	167	174	179	179
156	163	156	187	156
183	179	174	179	170
156	187	179	183	174
187	167	159	170	179

The engineer hypothesized that the mean Brinell hardness of *all* such ductile iron pieces is greater than 170. Therefore, he was interested in testing the hypotheses:

$$H_0 : \mu = 170$$

$$H_1 : \mu > 170$$

Help engineer to test hypothesis with 95% confidence.

2. For two samples with $n_1 = 25$ and $n_2 = 15$ from normally distributed populations it was found that $\bar{x} = 9.79, \bar{y} = 9.60$. Test hypothesis $H_0 : \mu_1 = \mu_2$ if $\sigma_1 = \sigma_2 = 0.3$, $\alpha = 0.01$.
3. There are two samples ($n_1 = n_2 = 9$) from total population with variance $\sigma^2 = 25$. Average values in samples are $\bar{x} = 2$ and $\bar{y} = 3$. Can the difference be explained by random causes, if error type I is 0.05?
4. Test hypothesis that two samples with equal variances from normally distributed populations have equal means, type I error is 0.05.

a)

X: 45, 48, 53, 44, 59, 60, 41, 43, 57;

Y: 51, 50, 42, 44, 39, 40, 48, 38, 59, 55, 51.

b)

X: 2.50; 2.50; 2.60; 2.75; 2.80; 2.80; 2.95;

Y: 2.50; 2.80; 2.85; 2.90; 2.90; 2.95; 3.40.

5. The only one device was made two series of measurements:

1) 2,5 3,2 3,5 3,8 3,5.

2) 2.0 2.7 2.5 2.9 2.3 2.6.

a) Assuming that measurements follow a normal distribution with equal variances, test the hypothesis of equality in means with $\alpha = 0.05$;

b) test the hypothesis that the variance is the same for these measurements, $\alpha=0.05$.

c) test the hypothesis that the variance for the first sample is 0.3, $\alpha = 0.05$.

6. $\bar{x} = 0,103$, $\bar{y} = 0,368$ – are sample means, where $n_1 = n_2 = 50$, population variance is 1. Test the hypothesis of equal sample means $H_0 : \mu_1 - \mu_2 = 1$ with $\alpha = 0.1$.

7. For two samples it is known that $n_1 = 11$, $n_2 = 15$, $S_1^2 = 0.76$ and $S_2^2 = 0.38$.

Assuming normal distribution in general population, test hypothesis about equality of variances with $\alpha = 0.1$.

8. A biologist was interested in determining whether sunflower seedlings treated with an extract from *Vinca minor* roots resulted in a lower average height of sunflower seedlings than the standard height of 15.7 cm. The biologist treated a random sample of $n = 33$ seedlings with the extract and subsequently obtained the following heights:

11.5	11.8	15.7	16.1	14.1	10.5
15.2	19.0	12.8	12.4	19.2	13.5
16.5	13.5	14.4	16.7	10.9	13.0
15.1	17.1	13.3	12.4	8.5	14.3
12.9	11.1	15.0	13.3	15.8	13.5
9.3	12.2	10.3			

The biologist's hypotheses are:

$$H_0 : \mu = 15.7$$

$$H_A : \mu < 15.7$$

Help biologist to test hypothesis with 95% confidence.

9. Two sample were received from one general population with normal distribution:

x_i	1	2	2.5	3
m_i	2	1	3	4
y_i	-1	2	2.5	3
m_i	3	2	1	2

Can one assume that the random variables have the same sample means ($\alpha=0.05$)?

10. A manufacturer claims that the thickness of the spearmint gum it produces is 7.5 one-hundredths of an inch. A quality control specialist regularly checks this claim. On one production run, he took a random sample of $n = 10$ pieces of gum and measured their thickness. He obtained:

7.65	7.60	7.65	7.70	7.55
7.55	7.40	7.40	7.50	7.50

The quality control specialist's hypotheses are:

$$H_0 : \mu = 7.5$$

$$H_A : \mu \neq 7.5$$

Help manufacturer to test hypothesis with 95% confidence.

11. Using site www.investing.com create a dataset of two series (EUR/USD, GBP/USD) for the last 8 month with daily observations. Test the following hypothesis ($\alpha=0.05$):

a) $H_0: \mu_1 = \mu_2$

b) $H_0: \mu_1 - \mu_2 = -0.15$

c) $H_0: \sigma_1 = \sigma_2$