

## Statistical distributions

### Examples with solutions:

1. Two balls are drawn at random in succession without replacement from an urn containing 4 red balls and 6 black balls. Find the probabilities of all the possible outcomes. What is the expected number of red balls?

**Answer:**

Possible Outcomes	RR	RB	BR	BB
$X$	2	1	1	0
$P$	2/15	4/15	4/15	5/15

$E(X)=0.8$ . This means that if we performed this experiment 1000 times, we would expect to get 800 red balls.

2. A jar of coffee is picked at random from a filling process in which an automatic machine is filling coffee jars each with 1 kg of coffee. Due to some faults in the automatic process, the weight of a jar could vary from jar to jar in the range 0.9 kg to 1.05 kg, excluding the latter. Let  $X$  denote the weight of a jar of coffee selected. What is the range of  $X$ ?

The weight of a jar of coffee selected is a continuous random variable. The following table gives the weight in kg of 100 jars recently filled by the machine. It lists the observed values of the continuous random variable and their corresponding frequencies. Find the probabilities for each weight category.

**Answer:**

$$(0.9 \leq X < 1.05).$$

Weight $X$	Number of Jars	Probability $P(a \leq X < b)$
0.900 - 0.925	1	0.01

0.925 - 0.950	7	0.07
0.950 - 0.975	25	0.25
0.975 - 1.000	32	0.32
1.000 - 1.025	30	0.30
1.025 - 1.050	5	0.05
<b>Total</b>	<b>100</b>	<b>1.00</b>

3. I throw a die and get \$1 if it is showing 1, and get \$2 if it is showing 2, and get \$3 if it is showing 3, etc. What is the amount of money I can expect if I throw it 100 times? (**\$350**)

4. The number of persons  $X$ , in a country family chosen at random has the following probability distribution:

$X$	1	2	3	4	5	6	7	8
$P(X)$	0.34	0.44	0.11	0.06	0.02	0.01	0.01	0.01

Find the average family size  $E(X)$ . (**2.1 people**)

5. In a card game with my friend, I pay a certain amount of money each time I lose. I win \$4 if I draw a jack or a queen and I win \$5 if I draw a king or ace from an ordinary pack of 52 playing cards. If I draw other cards, I lose. What should I pay so that we come out even? (That is, the game is "fair"?) (**I would need to pay \$2 for it to be a fair game**).

6. Find  $V(X)$  for the following probability distribution:

$X$	8	12	16	20	24
$P(X)$	81	61	83	41	121

**Answer:  $V(X) = 20$ .**

7. A die is tossed 3 times. What is the probability of
  - a) No fives turning up? (**0.5787**)
  - b) 1 five? (**0.34722**)
  - c) 3 fives? ( **$4.6296 \times 10^{-3}$** )
  
8. Hospital records show that of patients suffering from a certain disease, 75% die of it. What is the probability that of 6 randomly selected patients, 4 will recover? (**0.0329595**)
  
9. In the old days, there was a probability of 0.8 of success in any attempt to make a telephone call. (This often depended on the importance of the person making the call, or the operator's curiosity!). Calculate the probability of having 7 successes in 10 attempts. (**0.20133**)
  
10. A (blindfolded) marksman finds that on the average he hits the target 4 times out of 5. If he fires 4 shots, what is the probability of
  - a) more than 2 hits? (**0.8192**)
  - b) at least 3 misses? (**0.0272**)
  
11. A life insurance salesman sells on the average 3 life insurance policies per week. Use Poisson's law to calculate the probability that in a given week he will sell
  - a) Some policies (**0.95021**)
  - b) 2 or more policies but less than 5 policies. (**0.61611**)

- c) Assuming that there are 5 working days per week, what is the probability that in a given day he will sell one policy? **(0.32929)**

12. Twenty sheets of aluminum alloy were examined for surface flaws. The frequency of the number of sheets with a given number of flaws per sheet was as follows:

Number of flaws	Frequency
0	4
1	3
2	5
3	2
4	4
5	1
6	1

What is the probability of finding a sheet chosen at random which contains 3 or more surface flaws? **(0.40396)**

13. Find the following probabilities:

- a)  $P(Z > 1.06)$  **(0.1446)**
- b)  $P(Z < -2.15)$  **(0.0158)**
- c)  $P(1.06 < Z < 4.00)$  **(0.1446)**
- d)  $P(-1.06 < Z < 4.00)$  **(0.8554)**

14. It was found that the mean length of {100}100 parts produced by a lathe was 20.05 mm with a standard deviation of 0.02 mm. Find the probability that a part selected at random would have a length

- a) between 20.03 mm and 20.08 mm (**0.7745**)
- b) between 20.06 mm and 20.07 mm (**0.1498**)
- c) less than 20.01 mm (**0.0228**)
- d) greater than 20.09 mm (**0.0228**).

15. A company pays its employees an average wage of \$3.25 an hour with a standard deviation of 60 cents. If the wages are approximately normally distributed, determine

- a) the proportion of the workers getting wages between \$2.75 and \$3.69 an hour; (**0.566**)
- b) the minimum wage of the highest 5%. (**\$4.24**)

16. The average life of a certain type of motor is 10 years, with a standard deviation of 2 years. If the manufacturer is willing to replace only 3% of the motors because of failures, how long a guarantee should she offer? Assume that the lives of the motors follow a normal distribution. (**6.24 years**)

### Problems:

1. A coin is thrown twice. Describe the sample space. Let A – number of heads. Find the distribution of the random variable A, expected value and variance of A.
2. Dice is thrown twice. Describe the sample space. Let B – the sum of points on dice. Find the distribution of the random variable B, expected value of B.
3. The coin toss until the head falls. Describe the sample space. Let C – is number of tosses. Calculate: a) the distribution of the random variable C; b)  $P\{C > 1\}$ ,  $P\{C \leq n\}$ .
4. One shoots the target until first hit. The hit probability is equal to p by each shoot. Describe sample space. Let D is the number of shots taken. Calculate distribution of the random variable D.
5. What is the probability that by 5 coin tosses head will be observed from 2 to 4 times?

6. The probability of defective item is 0.02. What is the probability that among 100 items less than 3 defective ones will be found?
7. The coin was tossed 100 times. What is the probability that the total number of heads is in the range of 45 to 55?
8. The probability of hitting is 0.4. How many shots one should make to have at least one hit with probability at least 0.9?
9. The textbook has 50 errors in 500 pages. What is the probability that the section of 30 pages has: a) 2 errors? b) less than 2 errors? c) 2 or more errors? d) 0 errors?
10. On average one lamp fails among 200 lamps per month. There are 400 lamps. How likely is it that during a month fail: a) 3 lamps? b) at least 3 lamps? c) 0 lamps?
11. What is the probability that at 10 coin tosses head will be observed: a) from 4 to 6 times? b) from 3 to 5 times? c) 0 times? d) at least 4 times?
12. The company lacks in an average 5% of items from the catalogue. What is the probability that all 8 ordered items are present?
13. Dice is thrown 6 times. Find the probability that “3” or “6” will be observed twice.
14. The battery had 14 shots on the object, hit probability is 0.2. Calculate: a) the most likely number of hits and probability of this event; b) the probability that it was at least 4 hits.
15. The probability of hitting the target with every shot is 0.001. Find the probability of two or more hits if it was made 5,000 shots.
16. 1000 characters were transferred; each sign can be distorted with probability 0.004. Find the probability that less than 3 characters were distorted.
17. The probability of success in each trial is 0.25. How likely is it that in 300 trials will be successful: a) exactly 75 trials? b) exactly 85 trials?
18. The probability of failure of one unit equals 0.1. Determine the probability that from 100 devices fail: a) at least 20; b) less than 15; c) from 6 to 18 devices.
19. How many dice must be rolled to have at least a 95% chance of rolling a six? 99%? 99.9%?

Table 1. Function  $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ .

x	0	1	2	3	4	5	6	7	8	9
0,0	0,3989	3989	3989	3988	3986	3984	3982	3980	3977	3973
0,1	3970	3965	3961	3956	3951	3945	3939	3932	3925	3918
0,2	3910	3902	3894	3885	3876	3867	3857	3847	3836	3825
0,3	3814	3802	3790	3778	3765	3752	3739	3726	3712	3697
0,4	3683	3668	3653	3637	3621	3605	3589	3572	3555	3538
0,5	3521	3503	3485	3467	3443	3429	3410	3391	3372	3352
0,6	3332	3312	3292	3271	3251	3230	3209	3187	3166	3144
0,7	3123	3101	3079	3056	3034	3011	2989	2966	2943	2920
0,8	2897	2874	2850	2827	2803	2780	2756	2732	2709	2685
0,9	2661	2637	2613	2589	2565	2541	2516	2492	2468	2444
1,0	0,2420	2396	2371	2347	2323	2299	2275	2251	2227	2203
1,1	2179	2155	2131	2107	2083	2059	2036	2012	1989	1965
1,2	1942	1919	1895	1872	1849	1826	1804	1781	1758	1736
1,3	1714	1691	1669	1647	1626	1604	1582	1561	1539	1518
1,4	1497	1476	1456	1435	1415	1394	1374	1354	1334	1315
1,5	1295	1276	1257	1238	1219	1200	1182	1163	1145	1127
1,6	1109	1092	1074	1057	1040	1023	1006	0989	0973	0957
1,7	0940	0925	0909	0893	0878	0863	0848	0833	0818	0804
1,8	0790	0775	0761	0748	0734	0721	0707	0694	0681	0669
1,9	0656	0644	0632	0620	0608	0596	0584	0573	0562	0551
2,0	0,0540	0529	0519	0508	0498	0488	0478	0468	0459	0449
2,1	0440	0431	0422	0413	0404	0396	0387	0379	0371	0363
2,2	0355	0347	0339	0332	0325	0317	0310	0303	0297	0290
2,3	0283	0277	0270	0264	0258	0252	0246	0241	0235	0229
2,4	0224	0219	0213	0208	0203	0198	0194	0189	0184	0180
2,5	0175	0171	0167	0163	0158	0154	0151	0147	0143	0139
2,6	0136	0132	0129	0126	0122	0119	0116	0113	0110	0107
2,7	0104	0101	0099	0096	0093	0091	0088	0086	0084	0081
2,8	0079	0077	0075	0073	0071	0069	0067	0065	0063	0061
2,9	0060	0058	0056	0055	0053	0051	0050	0048	0047	0046
3,0	0,0044	0043	0042	0040	0039	0038	0037	0036	0035	0034
3,1	0033	0032	0031	0030	0029	0028	0027	0026	0025	0025
3,2	0024	0023	0022	0022	0021	0020	0020	0019	0018	0018
3,3	0017	0017	0016	0016	0015	0015	0014	0014	0013	0013
3,4	0012	0012	0012	0011	0011	0010	0010	0010	0009	0009
3,5	0009	0008	0008	0008	0008	0007	0007	0007	0007	0006
3,6	0006	0006	0006	0005	0005	0005	0005	0005	0005	0004
3,7	0004	0004	0004	0004	0004	0004	0003	0003	0003	0003
3,8	0003	0003	0003	0003	0003	0002	0002	0002	0002	0002
3,9	0002	0002	0002	0002	0002	0002	0002	0002	0001	0001

**Table 2. Standard normal table. Cumulative from mean (to Z).**  
**Cumulative probabilities for NEGATIVE z-values are shown in the following table:**

<b>z</b>	<b>.00</b>	<b>.01</b>	<b>.02</b>	<b>.03</b>	<b>.04</b>	<b>.05</b>	<b>.06</b>	<b>.07</b>	<b>.08</b>	<b>.09</b>
<b>-3.0</b>	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
<b>-2.9</b>	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
<b>-2.8</b>	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
<b>-2.7</b>	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
<b>-2.6</b>	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
<b>-2.5</b>	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
<b>-2.4</b>	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
<b>-2.3</b>	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
<b>-2.2</b>	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
<b>-2.1</b>	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
<b>-2.0</b>	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
<b>-1.9</b>	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
<b>-1.8</b>	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
<b>-1.7</b>	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
<b>-1.6</b>	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
<b>-1.5</b>	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
<b>-1.4</b>	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
<b>-1.3</b>	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
<b>-1.2</b>	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
<b>-1.1</b>	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
<b>-1.0</b>	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
<b>-0.9</b>	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
<b>-0.8</b>	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
<b>-0.7</b>	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
<b>-0.6</b>	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
<b>-0.5</b>	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
<b>-0.4</b>	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
<b>-0.3</b>	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
<b>-0.2</b>	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
<b>-0.1</b>	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
<b>0.0</b>	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



**Cumulative probabilities for POSITIVE z-values are in the following table:**

<b>z</b>	<b>.00</b>	<b>.01</b>	<b>.02</b>	<b>.03</b>	<b>.04</b>	<b>.05</b>	<b>.06</b>	<b>.07</b>	<b>.08</b>	<b>.09</b>
<b>0.0</b>	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
<b>0.1</b>	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
<b>0.2</b>	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
<b>0.3</b>	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
<b>0.4</b>	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
<b>0.5</b>	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
<b>0.6</b>	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
<b>0.7</b>	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
<b>0.8</b>	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
<b>0.9</b>	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
<b>1.0</b>	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
<b>1.1</b>	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
<b>1.2</b>	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
<b>1.3</b>	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
<b>1.4</b>	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
<b>1.5</b>	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
<b>1.6</b>	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
<b>1.7</b>	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
<b>1.8</b>	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
<b>1.9</b>	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
<b>2.0</b>	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
<b>2.1</b>	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
<b>2.2</b>	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
<b>2.3</b>	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
<b>2.4</b>	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
<b>2.5</b>	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
<b>2.6</b>	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
<b>2.7</b>	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
<b>2.8</b>	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
<b>2.9</b>	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
<b>3.0</b>	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990

Table 3. Poisson distribution  $p_k(\lambda) = \frac{\lambda^k}{k!} e^{-\lambda}$ .

	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1	2	3	4	5
0	0,90484	0,81873	0,74082	0,67032	0,60653	0,54881	0,49659	0,44933	0,40657	0,36788	0,13534	0,04979	0,01832	0,00674
1	0,09048	0,16375	0,22225	0,26813	0,30327	0,32929	0,34761	0,35946	0,36591	0,36788	0,27067	0,14936	0,07326	0,03369
2	0,00452	0,01637	0,03334	0,05363	0,07582	0,09879	0,12166	0,14379	0,16466	0,18394	0,27067	0,22404	0,14653	0,08422
3	0,00015	0,00109	0,00333	0,00715	0,01264	0,01976	0,02839	0,03834	0,04940	0,06131	0,18045	0,22404	0,19537	0,14037
4	0,00000	0,00005	0,00025	0,00072	0,00158	0,00296	0,00497	0,00767	0,01111	0,01533	0,09022	0,16803	0,19537	0,17547
5	0,00000	0,00000	0,00002	0,00006	0,00016	0,00036	0,00070	0,00123	0,00200	0,00307	0,03609	0,10082	0,15629	0,17547
6	0,00000	0,00000	0,00000	0,00000	0,00001	0,00004	0,00008	0,00016	0,00030	0,00051	0,01203	0,05041	0,10420	0,14622
7	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00001	0,00002	0,00004	0,00007	0,00344	0,02160	0,05954	0,10444
8	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00001	0,00086	0,00810	0,02977	0,06528
9	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00019	0,00270	0,01323	0,03627
10	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00004	0,00081	0,00529	0,01813
11	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00001	0,00022	0,00192	0,00824
12	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00006	0,00064	0,00343
13	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00001	0,00020	0,00132
14	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00006	0,00047
15	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00002	0,00016
16	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00005
17	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00001