

## Seminar 3. Conditional Probabilities

### Examples with answers:

1. The probabilities that three different archers, A, B hit the mark, independently of one another, are respectively  $\frac{1}{6}$ ,  $\frac{1}{4}$  and  $\frac{1}{3}$ . Everyone shoots an arrow. If only one hits the mark, what is the probability he is archer A? ( $\frac{6}{31}$ )
2. You toss a fair coin three times. Given that you have observed at least one heads, what is the probability that you observe at least two heads? ( $\frac{4}{7}$ )
3. A machine produces parts that are either good (90%), slightly defective (2%), or obviously defective (8%). Produced parts get passed through an automatic inspection machine, which is able to detect any part that is obviously defective and discard it. What is the quality of the parts that make it through the inspection machine and get shipped? (0.978)
4. Suppose that five good fuses and two defective ones have been mixed up. To find the defective fuses, we test them one-by-one, at random and without replacement. What is the probability that we are lucky and find both of the defective fuses in the first two tests? ( $\frac{1}{21}$ )
5. Two cards from an ordinary deck of 52 cards are missing. What is the probability that a random card drawn from this deck is a spade? ( $\frac{1}{4}$ )
6. 8 Urn 1 contains 5 white balls and 7 black balls. Urn 2 contains 3 whites and 12 black. A fair coin is flipped; if it is Heads, a ball is drawn from Urn 1, and if it is Tails, a ball is drawn from Urn 2. Suppose that this experiment is done and you learn that a white ball was selected. What is the probability that this ball was in fact taken from Urn 2? (i.e., that the coin flip was Tails) ( $\frac{12}{37}$ )
7. One half percent of the population has a particular disease. A test is developed for the disease. The test gives a false positive 3% of the time and a false negative 2% of the time. Joe just got the bad news that the test came back positive; what is the probability that Joe has the disease? (0.14)

## Problems

1. Throw two dice. What is the probability that at least one “3” falls, knowing that the sum is 7?
2. Throw three dice. What is the probability that at least one “6” falls if all three dice have different points?
3. A coin is tossed three times. What is the probability that exactly two heads occur, given that (a) the first outcome was a head? (b) the first outcome was a tail? (c) the first two outcomes were heads? (d) the first two outcomes were tails? (e) the first outcome was a head and the third outcome was a head?
4. A die is rolled twice. What is the probability that the sum of the faces is greater than 7, given that (a) the first outcome was a 4? (b) the first outcome was greater than 3? (c) the first outcome was a 1? (d) the first outcome was less than 5?
5. There are  $m$  white and  $n$  black balls in the box, two balls are chosen randomly. It is known that the first ball is white. What is the probability that the second ball is also white?
6. It is known that when throwing dice 10 times at least one “1” was observed. What is the probability that two or more “1” be observed?
7. It is known that 5% of men and 0.25% of all women - colour-blind. Randomly chosen person is colour-blind. How likely is it that this is a man? (Consider the equal number of men and women).
8. Two hunters strike the goal with probabilities of 0.7 and 0.8 respectively. Each of them makes one shot. What is the probability that a) both strike? b) no one strikes? c) at least one strikes? d) Only one strikes a target?
9. Two players in turn throw a coin. The winner is the one who first rolled head. Find the probability of winning for each player.
10. Hunter A hits the target with probability  $p_1 = 0.6$ , shooter B - with a probability  $p_2 = 0.5$ , shooter C – with probability  $p_3 = 0.4$ . Hunters made one shoot each. It is known that there are two hits. What's more likely: C hit the target or not?

11. Three hunters once had one shot at the bear each. The bear was killed by a single bullet. What is the probability that a bear was killed by first, second or third hunter, if hit probabilities for them are  $p_1 = 0.2$ ,  $p_2 = 0.4$ ,  $p_3 = 0.6$ ?
12. Production level of the plant A is twice bigger than plant B. Manufacturing defect level of the first plant is  $p_1$ , of the second -  $p_2$ . Randomly taken item is defective. What is the probability that it was produced by B?
13. Two archers independently make one shot on target. The probability of hitting for them 0.8 and 0.4 respectively. We know for sure that there is only one hit. Find the probability that the target was hit by the first shooter.
14. The factory made screws. The first machine produces 25%, the second - 35%, the third - 40% of all products. Defective ratios are 5, 4 and 2% respectively.
  - a. What is the probability that randomly selected screw is defective?
  - b. Randomly selected screw was defective. What is the probability that it was produced by first, second, third machine?
15. A doctor assumes that a patient has one of three diseases  $d_1$ ,  $d_2$ , or  $d_3$ . Before any test, he assumes an equal probability for each disease. He carries out a test that will be positive with probability 0.8 if the patient has  $d_1$ , 0.6 if he has disease  $d_2$ , and 0.4 if he has disease  $d_3$ . Given that the outcome of the test was positive, what probabilities should the doctor now assign to the three possible diseases?
16. A student is applying to Harvard and Dartmouth. He estimates that he has a probability of 0.5 of being accepted at Dartmouth and 0.3 of being accepted at Harvard. He further estimates the probability that he will be accepted by both is 0.2. What is the probability that he is accepted by Dartmouth if he is accepted by Harvard? Is the event "accepted at Harvard" independent of the event "accepted at Dartmouth"?