

## Seminar 2. Theory of Probabilities

### Examples with answers

1. A die is rolled, find the probability that an even number is obtained. (1/2)
2. Two coins are tossed, find the probability that two heads are obtained. (1/4)
3. Two dice are rolled, find the probability that the sum is equal to 4 (1/12)
4. A die is rolled and a coin is tossed, find the probability that the die shows an odd number and the coin shows a head. (1/4)
5. A card is drawn at random from a deck of cards. Find the probability of getting the 3 of diamond. (1/52)
6. A card is drawn at random from a deck of cards. Find the probability of getting a queen. (1/13)
7. A jar contains 3 red marbles, 7 green marbles and 10 white marbles. If a marble is drawn from the jar at random, what is the probability that this marble is white? (1/2)
8. The blood groups of 200 people is distributed as follows: 50 have type A blood, 65 have B blood type, 70 have O blood type and 15 have type AB blood. If a person from this group is selected at random, what is the probability that this person has O blood type? (0.35)

### Problems

1. Dice is thrown twice. Describe the sample space. Describe events: A - sum of points on two dice equals 8; B - at least once falls "6".
2. Throw a coin and dice. Describe the sample space.
3. Throw a coin until it falls head. Describe the sample space.
4. Build a set of elementary events when throwing a coin until head falls twice.
5. Simplify sets:
  - a.  $(A \cup B) \cup (A \cup \bar{B})$ ?
  - b.  $(A \cup B) \cap (\bar{A} \cup B) \cap (A \cup \bar{B})$ ?
  - c.  $(A \cup B) \cap (B \cup C)$ .

6. Prove expressions:

- a.  $(A \cup B) \setminus B = A \setminus (A \cap B) = A \cap \bar{B}$ ;
- b.  $(A \cup B) \setminus (A \cap B) = (A \cap \bar{B}) \cup (\bar{A} \cap B)$ ;
- c.  $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ ;
- d.  $(A \cup B) \cap (A \cup \bar{B}) \cup (\bar{A} \cup B) \cap (\bar{A} \cup \bar{B}) = \Omega$ ;
- e.  $(A \cup B) \cap (A \cup \bar{B}) \cap (\bar{A} \cup B) \cap (\bar{A} \cup \bar{B}) = \emptyset$ ;
- f.  $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$ ;
- g.  $A \setminus (B \cup C) = (A \setminus B) \setminus C$ .

7. A card is drawn at random from a deck of cards. Find the probability of getting the King of hearts.
8. There are R students in the group. What is the probability that at least two of them have the same birthday?
9. Dice tossed six times. Calculate the probability that all six faces fall out.
10. There are 7 passengers in the elevator. Elevator stops at 10 floors. What is the probability that two passengers never go out on one floor?
11. Calculate the probability that the birthdays of 12 people are in different months of the year.
12. In an urn there are 10 balls, 3 white and 7 black. One randomly removed one ball. How likely is it that the ball is: a) white? b) black?
13. In an urn there are 10 balls, 3 white and 7 black. What is the probability that the randomly drawn 2 balls are: a) black? b) white?
14. During the exam it can be offered N questions. The student knows the answers to n questions. The examiner asks the student k questions, but in order to pass the exam, student must answer at least on r ( $r < k$ ). What is the probability that a student will pass the exam?
15. Let C and D be two events with  $P(C) = 0.25$ ,  $P(D) = 0.45$ , and  $P(C \cap D) = 0.1$ . What is  $P(C^c \cap D)$ ?
16. Throw 12 dice. What is the probability that each number 1, 2, ..., 6 fall twice?

17. Distribution of bank depositors is the following:

Age	Deposit		
	< 1000\$	1000—5000 \$	> 5000\$
< 30 years	5%	15%	8%
30—50 years	8%	25%	20%
> 50 years	7%	10%	2%

Let A and B are the following events:

$A = \{ \text{randomly selected customer has a deposit at least } \$ 5000 \}$

$B = \{ \text{randomly selected customer is over 30 years} \}$ .

Define:  $P(A)$ ,  $P(B)$ ,  $P(A \cup B)$ ,  $P(A \cap B)$ .

18. Analysis of 10,000 purchases in the supermarket gave such results:

Payments	Good			
	Women's clothing	Men's clothing	Sport goods	Household goods
Cash	6%	9%	3%	7%
Credit card	41%	9%	22%	3%

Let A, B, C, D following events:

$A = \{ \text{randomly selected purchase paid by credit card} \}$

$B = \{ \text{random selected purchase is women's clothing} \}$

$C = \{ \text{randomly selected purchase is men's clothing} \}$

$D = \{ \text{randomly selected purchase is sporting goods} \}$ .

Compute  $P(A)$ ,  $P(B \cap A)$ ,  $P(A \cap D)$ ,  $P(A \cup B)$ ,  $P(A \cup C)$ .

19. A coin of diameter  $d$  is randomly thrown on the parquet floor. Parquet contains squares with side  $a$  ( $a > d$ ). What is the probability that the coin does not cross any line of parquet squares?

20. In one box there are 5 white and 10 black balls, the second contains 10 white and 5 black balls. Find the probability that at least one white ball was checked out, if only one ball was checked out from each box.

21. In the box there 10 red and 6 blue balls. One drew randomly two balls. What is the probability that the balls have the same colour?

22. Urn A has 3 black balls and 6 white balls. Urn B has 400 black balls and 400 white balls. Urn C has 6 black balls and 3 white balls. A person first randomly chooses one of the urns and then grabs a ball randomly from the chosen urn. What is the probability that the ball be black?

23. A jar contains 30 red marbles, 12 yellow marbles, 8 green marbles and 5 blue marbles. What is the probability that you draw and replace marbles 3 times and you get NO red marbles?

24. A jar contains 30 red marbles, 12 yellow marbles, 8 green marbles and 5 blue marbles. What is the probability that you draw and replace marbles 3 times and you get at least 1 Red?

25. A business office orders paper supplies from one of three vendors, V1, V2, or V3. Orders are to be placed on two successive days, one order per day. Thus, (V2, V3) might denote that vendor V2 gets the order on the first day and vendor V3 gets the order on the second day.

- a. List the sample points in this experiment of ordering paper on two successive days.
- b. Assume the vendors are selected at random each day and assign a probability to each sample point.
- c. Let A denote the event that the same vendor gets both orders and B the event that V2 gets at least one order. Find  $P(A)$ ,  $P(B)$ ,  $P(A \cup B)$  and  $P(A \cap B)$  by summing the probabilities of the sample points in these events.