

Seminar 1. Basics of Combinatorics

Examples with answers

1. How many 10-letter patterns can be formed from the letters of the word "BASKETBALL"? (604800)
2. A box contains 12 black and 8 green marbles. How many ways can 3 black and 2 green marbles be chosen? (248)
3. A Club consists of 20 members, of which 9 are male and 11 are female. Seven members will be selected to form an event-planning committee. How many committees of 4 females and 3 males can be formed? (${}_{11}C_4 \times {}_9C_3$)
4. How many 7-digit telephone numbers can be formed if the first digit cannot be 0 or 1? ($8 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$)
5. How many different 5-digit street addresses can have the digits 4, 7, 3, 4, and 8? (60)
6. Three hardcover books and 5 paperbacks are placed on a shelf. How many ways can the books be arranged if all the hardcover books must be together and all the paperbacks must be together? ($2! \times 3! \times 5!$)
7. How many permutations are there of the word "SCHOOL"? (360)
8. How many ways can you choose 4 groups of 4 people from 16 people, assuming the groups are distinct? (${}_{16}C_4 \times {}_{12}C_4 \times {}_8C_4 \times {}_4C_4$)
9. How many ways can you arrange 16 people into 4 rows of 4 desks each? (16!)
10. How many ways can you pair up 8 boys and 8 girls? (64)
11. In how many ways can a party of 4 men and 4 women be seated at a circular table so that no two women are adjacent? (Answer: The 4 men can be seated at the circular table such that there is a vacant seat between every pair of men in $(4-1)! = 3!$ Ways. Now 4 vacant seats can be occupied by 4 women in $4!$ Ways. Hence the required number of seating arrangements = $3! \times 4! = 144$)
12. Out of 2 Women and 5 Men, a committee of 3 is to be formed. In how many ways can it be formed if at least one woman is to be included? (25)

Problems:

1. One can reach city Q from city P by sea, air and road. Suppose that there are 2 ways by sea, 3 ways by air and 2 ways by road. Calculate total number of ways from P to Q by sea, air or road.
2. In the luggage office a combination lock has four digits. How many different combinations you can make with the numbers 1, 2, 3, 4, 5, if:
 - a. numbers in the code can be repeated?
 - b. numbers in the code are not repeated?
 - c. code begins with "3"?
 - d. code is an even number?
 - e. code is even number; numbers are not repeated?
3. How many three-digit numbers can be recorded with numbers 0, 1, 2, 3, 4?
4. By how many ways can 7 people stand in line to the cashier?
5. How many ways exist to distribute 15 different items between the three entities so that each person has 5 subjects?
6. Urn contains 10 black and 6 white balls, one selects 2 black and 3 white balls. In how many ways it can be done?
7. A student must pass 4 different exams for 8 days. In how many ways it can be done?
8. Six boxes of materials should be delivered to the eight floor construction. How many ways are there for distributing materials among floors? In how many of them at least two materials are delivered on the eighth floor?
9. Elevator with 9 passengers stops on ten floors. Passengers go out by groups of two, three or four persons. In how many ways it can be done?
10. There are 7 boys and 3 girls in one room. In how many ways can they be arranged in a row so that
 - a. the 3 girls form a single block (i.e. there is no boy between any two of the girls)?
 - b. the two end-positions are occupied by boys and no girls are adjacent?

11. In how many ways can 5 boys and 3 girls be seated around a table if
 - a. there is no restriction?
 - b. boy B1 and girl G1 are not adjacent?
 - c. no girls are adjacent?
12. Find the number of ways to seat n married couples around a table in each of the following cases:
 - a. Men and women alternate;
 - b. Every woman is next to her husband.
13. In how many ways can a committee of 5 be formed from a group of 11 people consisting of 4 teachers and 7 students if
 - a. there is no restriction in the selection?
 - b. the committee must include exactly 2 teachers?
 - c. the committee must include at least 3 teachers?
 - d. a particular teacher and a particular student cannot be both in the committee?
14. A 4-storey house is to be painted by some 6 different colours such that each storey is painted in one colour. How many ways are there to paint the house?
15. Ten chairs have been arranged in a row. Seven students are to be seated in seven of them so that no two students share a common chair. Find the number of ways this can be done if no two empty chairs are adjacent.
16. Eight boxes are arranged in a row. In how many ways can five distinct balls be put into the boxes if each box can hold at most one ball and no two boxes without balls are adjacent?
17. A group of 20 students, including 3 particular girls and 4 particular boys, are to be lined up in two rows with 10 students each. In how many ways can this be done if the 3 particular girls must be in the front row while the 4 particular boys be in the back?
18. In how many ways can 7 boys and 2 girls be lined up in a row such that the girls must be separated by exactly 3 boys?

19. In a group of 15 students, 3 of them are female. If at least one female student is to be selected, in how many ways can 7 students be chosen from the group
- to form a committee?
 - to take up 7 different posts in a committee?
20. Six scientists are working on a secret project. They wish to lock up the documents in a cabinet so that the cabinet can be opened when and only when three or more of the scientists are present. What is the smallest number of locks needed? What is the smallest number of keys each scientist must carry?
21. A 10-storey building is to be painted with some 4 different colours such that each storey is painted with one colour. It is not necessary that all 4 colours must be used. How many ways are there to paint the building, if
- there are no other restrictions?
 - any 2 adjacent stories must be painted with different colours?
22. Ten chairs have been arranged in a row. Seven students are to be seated in seven of them so that no two students share a common chair. Find the number of ways this can be done if no two empty chairs are adjacent.
23. How many 9-digit telephone numbers can be compiled from the digits 0,1,2,...,8,9 that no digit is repeated?
24. How many ways are there to arrange 9 books on a shelf?
25. In how many ways can 7 shuttle vans line up at the airport?
26. How many ways can we buy 13 chocolates?
27. In the class are 16 pupils. How many opportunities have the teacher if he wants choose two pupils randomly who will weeklies?
28. 5 heroes galloping on 5 horses behind. How many ways can sort them behind?
29. How many five-digit PIN - code can we create using the even numbers?
30. How many different possible 4 digits pins can be found on the 10-digit keypad?