KSE

Kyiv
School of
Economics

Econometric Tests

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Agenda

- Econometric tests
- Hypothesis testing example

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Linear regression

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \ldots + \beta_{k-1} x_{k-1t} + \varepsilon_t, t = \overline{1,n}$$

y_t - dependent variable;

$x_{1t}, x_{2t}, \ldots, x_{k-1t}$ - independent variables;

ε_t - residuals.

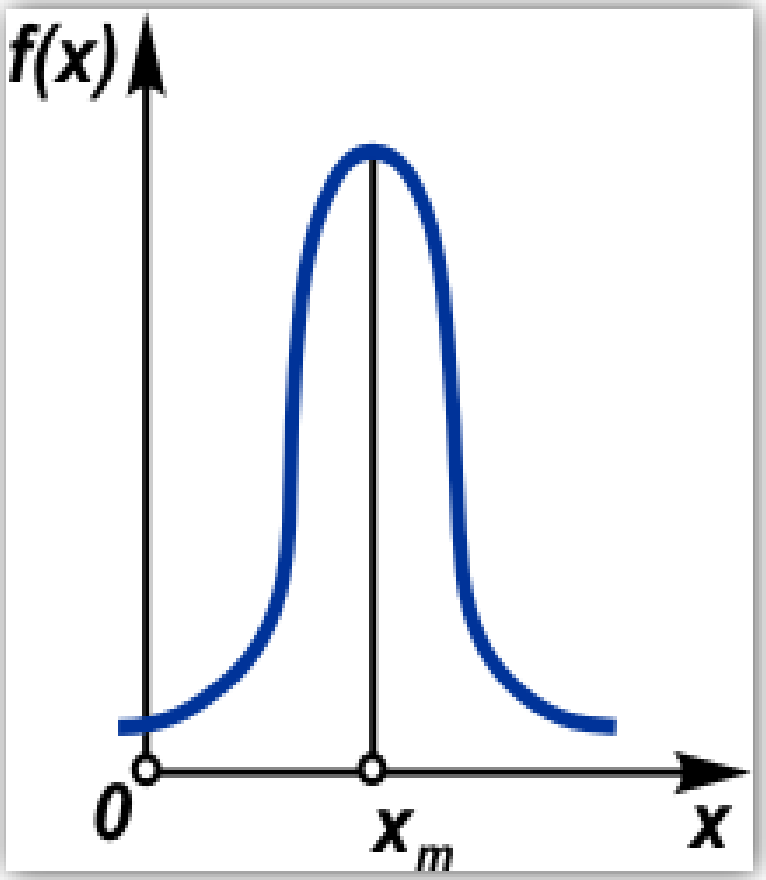
Multiple Regression Tests

- Test residual for normality
- Test parameter significance
  - Overall model
  - Individual coefficients
- Test for multicollinearity
- Test for model stability
- Test for residuals autocorrelation
- Test for residuals homoscedasticity
- Test for specification
- Test for stationary process

Test residual for normality

Check normality of residuals:

- Jarque-Bera statistics
- Shapiro–Wilk test



Jarque-Bera statistics

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}\left((K-3)^2 \right) \right)$$

$$S = \frac{\hat{\mu}_3}{\hat{\sigma}^3} = \frac{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^3}{\left(\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2\right)^{3/2}}$$

$$K = \frac{\hat{\mu}_4}{\hat{\sigma}^4} = \frac{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^4}{\left(\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2\right)^{2}}$$

- S is the sample skewness,
- K is the sample kurtosis.

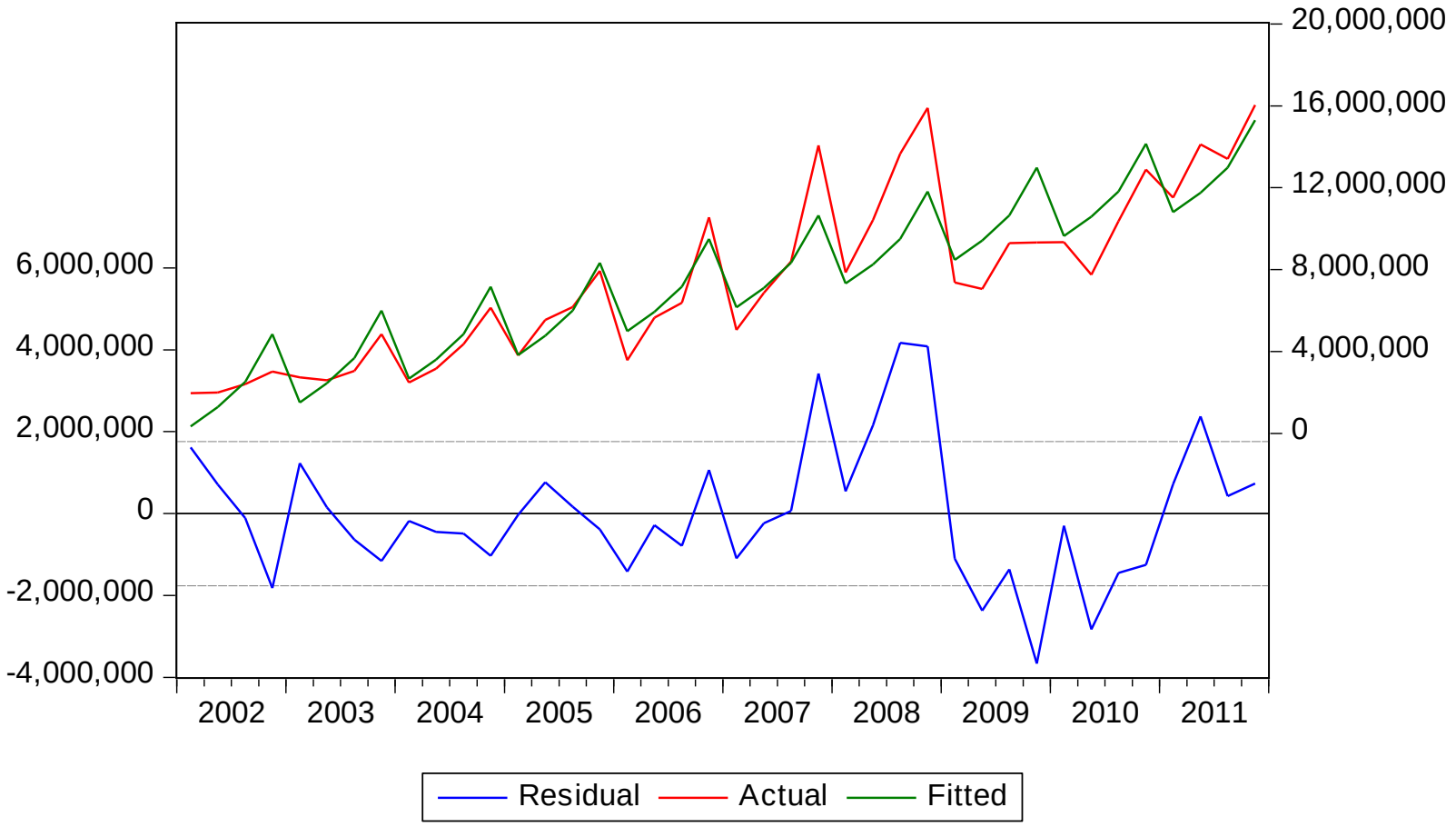
Example

Dependent Variable: TAX_ENT
Method: Least Squares
Date: 12/09/12 Time: 20:49
Sample: 2002Q1 2011Q4
Included observations: 40

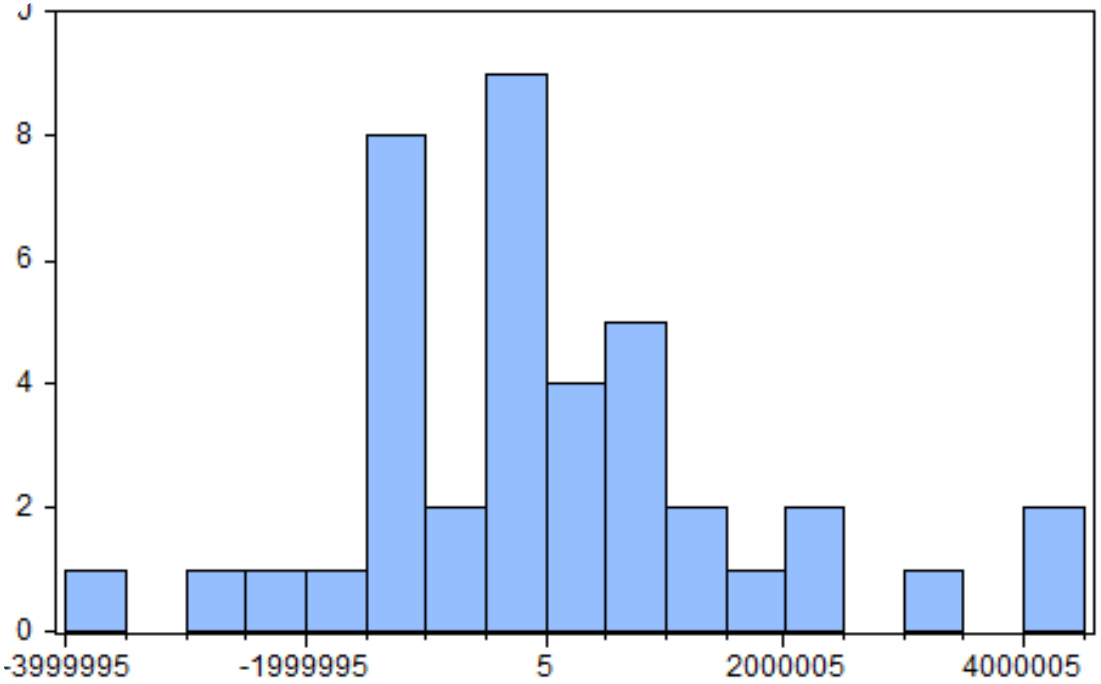
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3973770.	754540.7	5.266475	0.0000
@TREND	290525.1	24239.34	11.98568	0.0000
@SEAS(1)	-3627516.	791034.8	-4.585786	0.0001
@SEAS(2)	-2975920.	789175.7	-3.770922	0.0006
@SEAS(3)	-2032456.	788058.1	-2.579068	0.0143

R-squared	0.837415	Mean dependent var	7480035.
Adjusted R-squared	0.818834	S.D. dependent var	4138083.
S.E. of regression	1761318.	Akaike info criterion	31.71749
Sum squared resid	1.09E+14	Schwarz criterion	31.92860
Log likelihood	-629.3498	Hannan-Quinn criter.	31.79382
F-statistic	45.06800	Durbin-Watson stat	1.123746
Prob(F-statistic)	0.000000		

Residuals


20,000,000
16,000,000
12,000,000
8,000,000
4,000,000
0
6,000,000
4,000,000
2,000,000
0
-2,000,000
-4,000,000
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
Residual
Actual
Fitted

Check for normality



Series: Residuals	
Sample 2002Q1 2011Q4	
Observations 40	
Mean	-3.49e-11
Median	-204126.5
Maximum	4171075.
Minimum	-3659494.
Std. Dev.	1668551.
Skewness	0.590368
Kurtosis	3.696860
Jarque-Bera	3.132917
Probability	0.208783

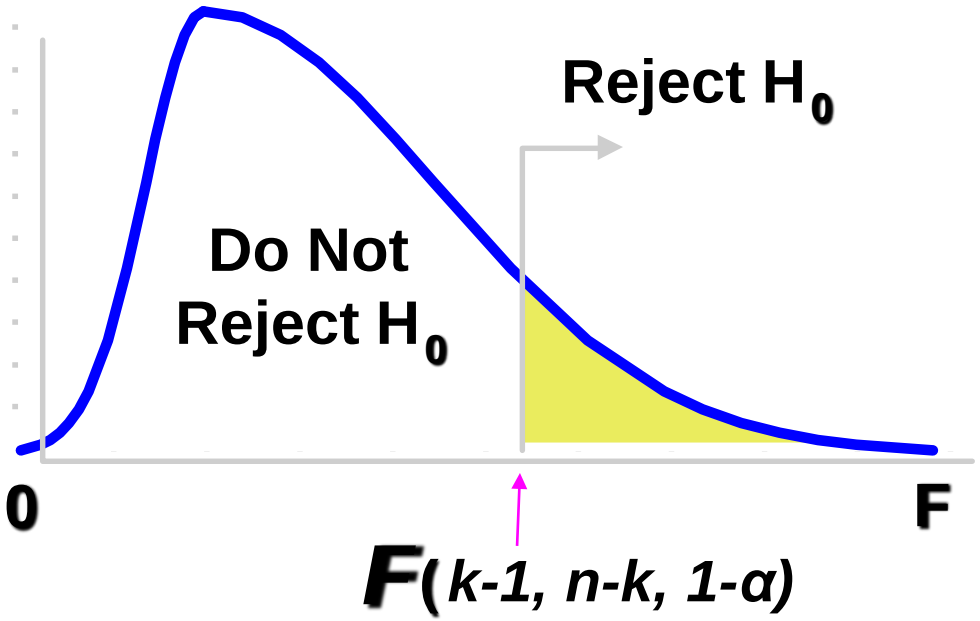
Test parameter significance: Overall model

- Hypotheses
  - H_0: $\beta_1 = \beta_2 = \ldots = \beta_{k-1} = 0$
    - No Linear Relationship
  - H_a: At Least One Coefficient Is Not 0
    - At Least One X Variable linearly Affects Y

$$F = \frac{RSS/(k-1)}{ESS/(n-k)} = \frac{R^2/(k-1)}{(1-R^2)/(n-k)} \overset{H_0}{\sim} F_{k-1,n-k}$$

Overall Significance Rejection Rule

- Reject H_0 in favor of H_a if F_{calc} falls in colored area



- Reject H_0 for H_a if P-value = $P(F>F_{calc})<\alpha$

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Prob(F-statistic)	0.000000		

Test of slope coefficients

- Hypothesis
  - H_0: β_i=m
  - H_a: β_i≠m

Slope Coefficient Test Statistic

$$t = \frac{\hat{\beta}_i - m}{S_{\hat{\beta}_i}}$$

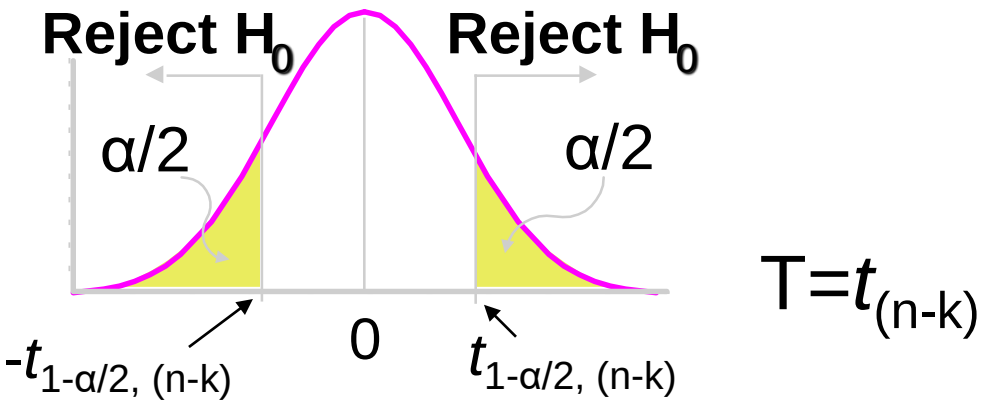
$$\text{where} \quad S_{\hat{\beta}_i} = \frac{S}{\sqrt{\sum_{i=1}^{n} X_i^2 - \frac{\left(\sum_{i=1}^{n} X_i\right)^2}{n}}}$$

$$\text{with} \quad S = \hat{\sigma} = \sqrt{\frac{RSS}{n-k}}$$

$$\text{and} \quad RSS = \sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2 = \sum_{i=1}^{n}\left[Y_i - \left(\hat{\beta}_0 + \sum_{i=1}^{k-1}\hat{\beta}_i X_i\right)\right]^2$$

Test of Slope Coefficient Rejection Rule

- Reject H_0 in favor of H_a if t falls in colored area



- Reject H_0 for H_a if P-value = P(T>|t|) < α

Special case: significance of coefficient

- Hypothesis
  - H_0: $\beta_i=0$
  - H_a: $\beta_i\neq 0$

$$t = \frac{\hat{\beta}_i}{S_{\hat{\beta}_i}}$$

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Example

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Prob(F-statistic)	0.000000		

Wald test

Null Hypothesis: $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$

Alternative hypothesis $H_1 : \beta_1$ or β_2 or β_3

or any two of them or all are nonzero.

At least one of them is significant.

In matrix notation

Hypothesis: $$Rb = r \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Test statistics with J numbers of restriction

$$F = \frac{(Rb - r)' \left[R \operatorname{cov}(b) R' \right]^{-1} (Rb - r)}{\frac{J}{\frac{RSS}{n-k}}}$$

Calculate F and compare it with the critical values F(J, n-k) from the Table.

Test for multicollinearity

- High correlation between X variables
- Coefficients measure combined effect
- Leads to unstable coefficients depending on X variables in model
- Always exists; matter of degree
- Example: Using both total number of rooms and number of bedrooms as explanatory variables in same model

Detecting Multicollinearity

- Farrar-Glauber Multicollinearity
- VIF-test

- Few remedies
  - Obtain new sample data
  - Eliminate one correlated X variable
  - Standardize your independent variables.

Example

$$\hat{s}_t = 0.4 + 0.8y_t + 0.2li_t - 0.1si_t$$

$$(0.9)\ \ (1.2)\ \ \ (0.4)\ \ \ \ (0.1)$$

$\bar{R}^2 = 0.98$, (standard errors in parentheses)

$(n = 60)$. where:

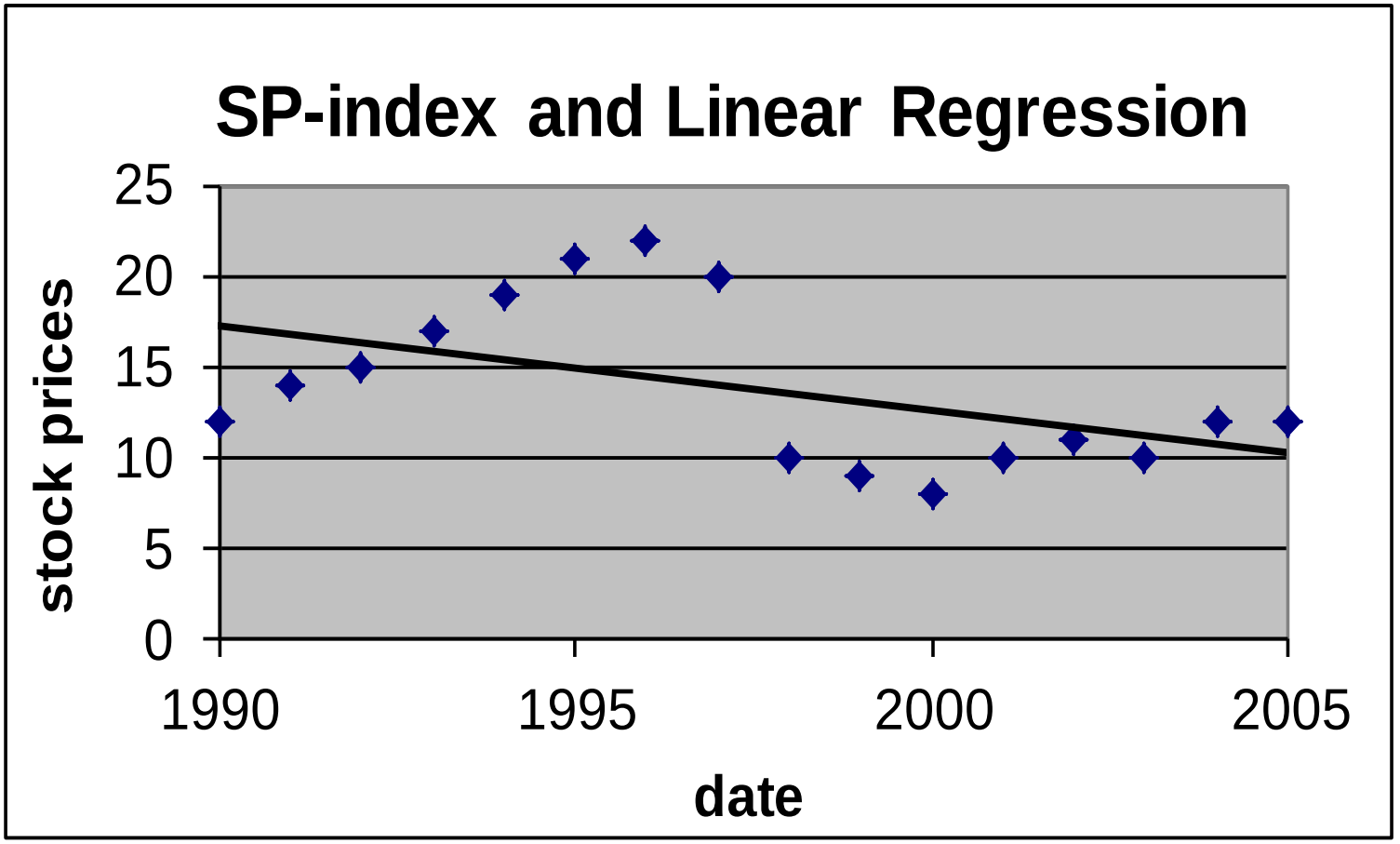
s_t – stock prices

y_t – output

li_t – long-run interest rates

si_t -short-run interest rates

Test for structural breaks



Chow Test

- Tests whether the coefficients in two linear regressions on different data sets are equal.

$$F = \frac{RSS_c - (RSS_1 + RSS_2)/k}{(RSS_1 + RSS_2)/n - 2k} \sim F_{k,n-2k}$$

$RSS_c - combined\_RSS$

$RSS_1 - pre - break\_RSS$

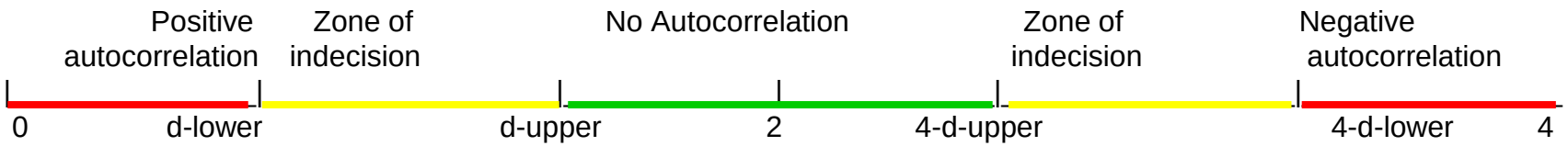
$RSS_2 - post - break\_RSS$

Test for residuals autocorrelation

- Durbin-Watson test (only checks for first order serial correlation in residuals)
- Breusch-Godfrey Test (checks for higher order autocorrelation AR(q) in residuals)

Durbin-Watson statistic

$$d = \frac{\sum (e_i - e_{i-1})^2}{\sum e_i^2}, \text{ for n and K - 1 d.f.}$$



- Autocorrelation is clearly evident
- Ambiguous – cannot rule out autocorrelation
- Autocorrelation in not evident

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Breusch-Godfrey Test

Higher Order Autocorrelation model : AR(p)

$$\mu_t = \rho_1 \mu_{t-1} + \rho_2 \mu_{t-2} + ... + \rho_p \mu_{t-p} + \varepsilon_t$$

Null Hypothesis

$$H_0 : \rho_1 = \rho_2 = ... = \rho_p = 0$$

Test Model :

$$\hat{\mu}_t = \delta_1 + \delta_2 X_{2t} + ... + \delta_k X_{kt} + \lambda_1 \hat{\mu}_{t-1} + ... + \lambda_p \hat{\mu}_{t-p} + \omega_t$$

Test Statistic

$$LM = (n - p) * R^2_{aux} \sim \chi^2_p$$

Tests for Heteroskedasticity

- There are two types of tests:
  - Tests for continuous changes in variance: White test, Breusch–Pagan tests, etc.
  - Tests for discrete (lumpy) changes in variance: the Goldfeld–Quandt test

Divide the n observations into h groups, of sizes $n_1..n_h$

Choose two groups, say 1 and 2.

$$H_0 : \sigma_1^2 = \sigma_2^2 \quad \text{against } H_a : \sigma_1^2 \neq \sigma_2^2$$

Test for specification

$$F^{k}_{n-m-k+1} \sim \frac{\frac{R_1^2 - R_0^2}{k}}{\frac{1-R_1^2}{n-m-k}}$$

Stationary process

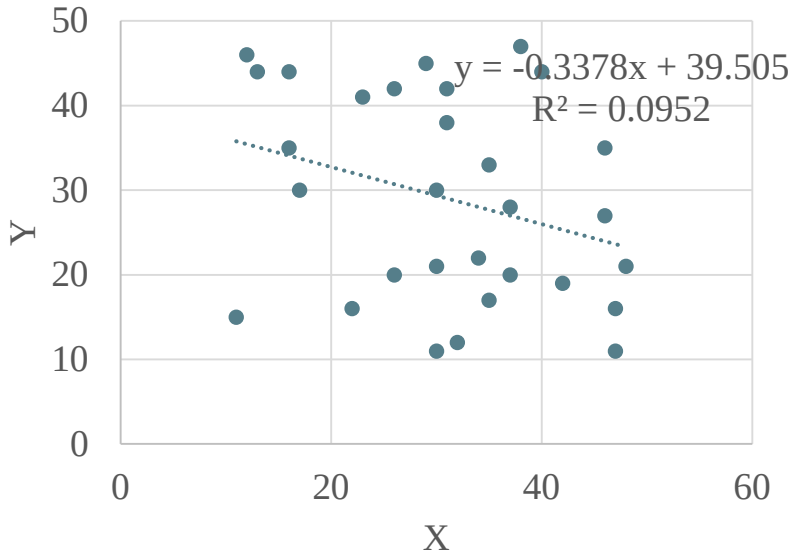
- A stationary process is a stochastic process whose joint probability distribution does not change when shifted in time.
- Parameters such as the mean and variance, if they are present, also do not change over time and do not follow any trends.

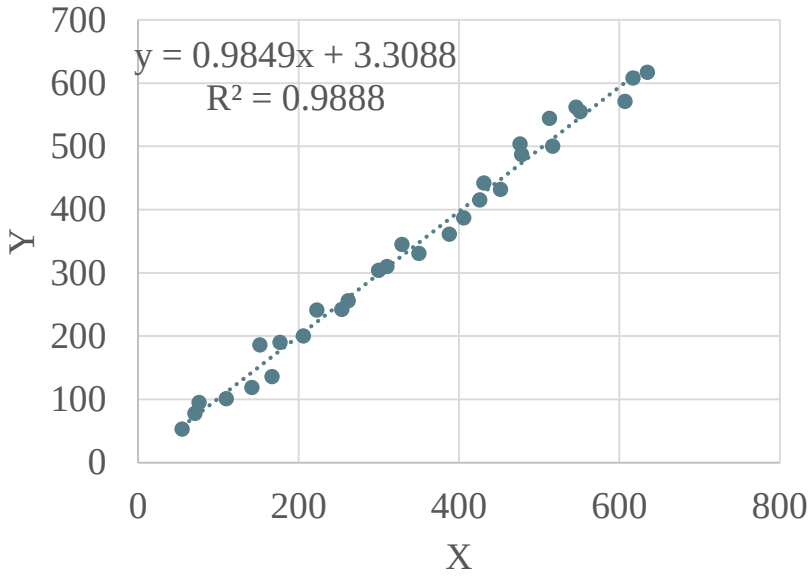
Solutions:

- Taking differences (Dickey-Fuller test)
- Trend-stationary processes

Why stationarity is important?

№	X	Y	T	X*	Y*
1	35	33	20	55	53
2	31	38	40	71	78
3	16	35	60	76	95
4	30	21	80	110	101
5	42	19	100	142	119
6	47	16	120	167	136
7	12	46	140	152	186
8	17	30	160	177	190
9	26	20	180	206	200
10	23	41	200	223	241
11	34	22	220	254	242
12	22	16	240	262	256
13	40	44	260	300	304
14	30	30	280	310	310
15	29	45	300	329	345
16	30	11	320	350	331
			…		
30	35	17	600	635	617





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Question

“What should we do, if we fail to find an appropriate model that satisfy all tests?”

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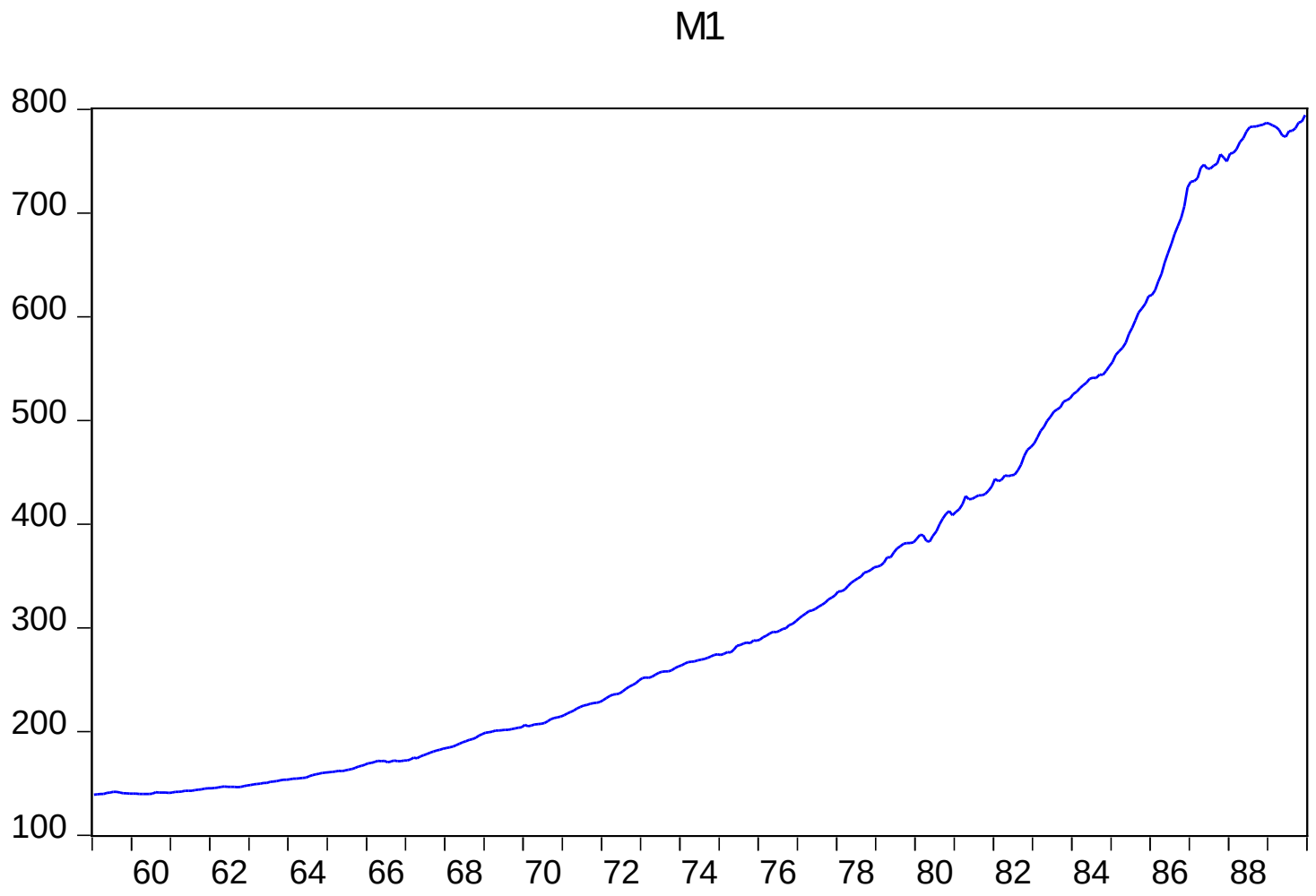
Hypothesis testing Example

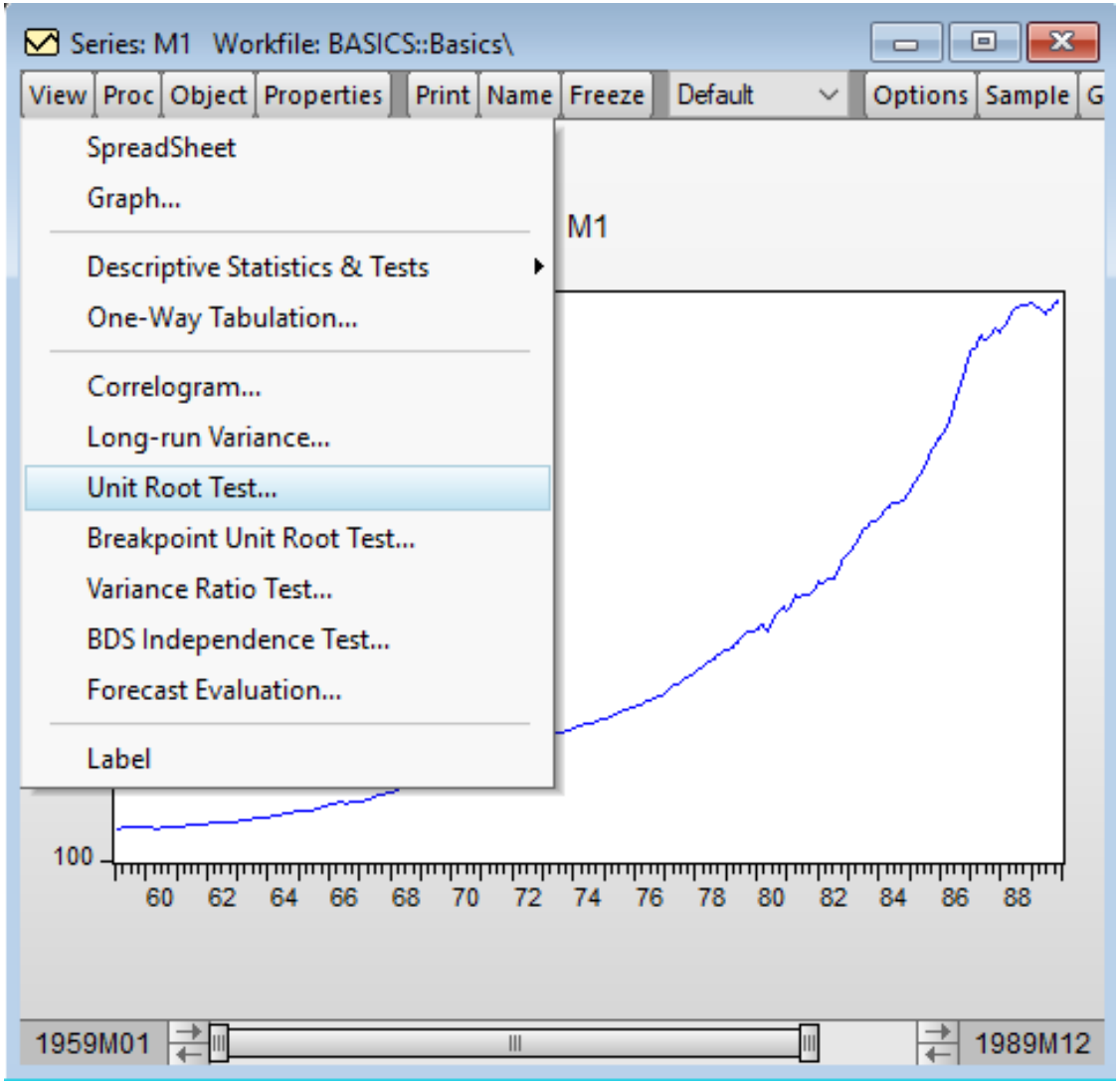
File: Basics.wf1

- M1 – Money stock: M1 (BIL$,SA)
- IP - Industrial production: total index (1987=100,SA)
- Interest rate: U.S.Treasury bills (% PER ANN,NSA)

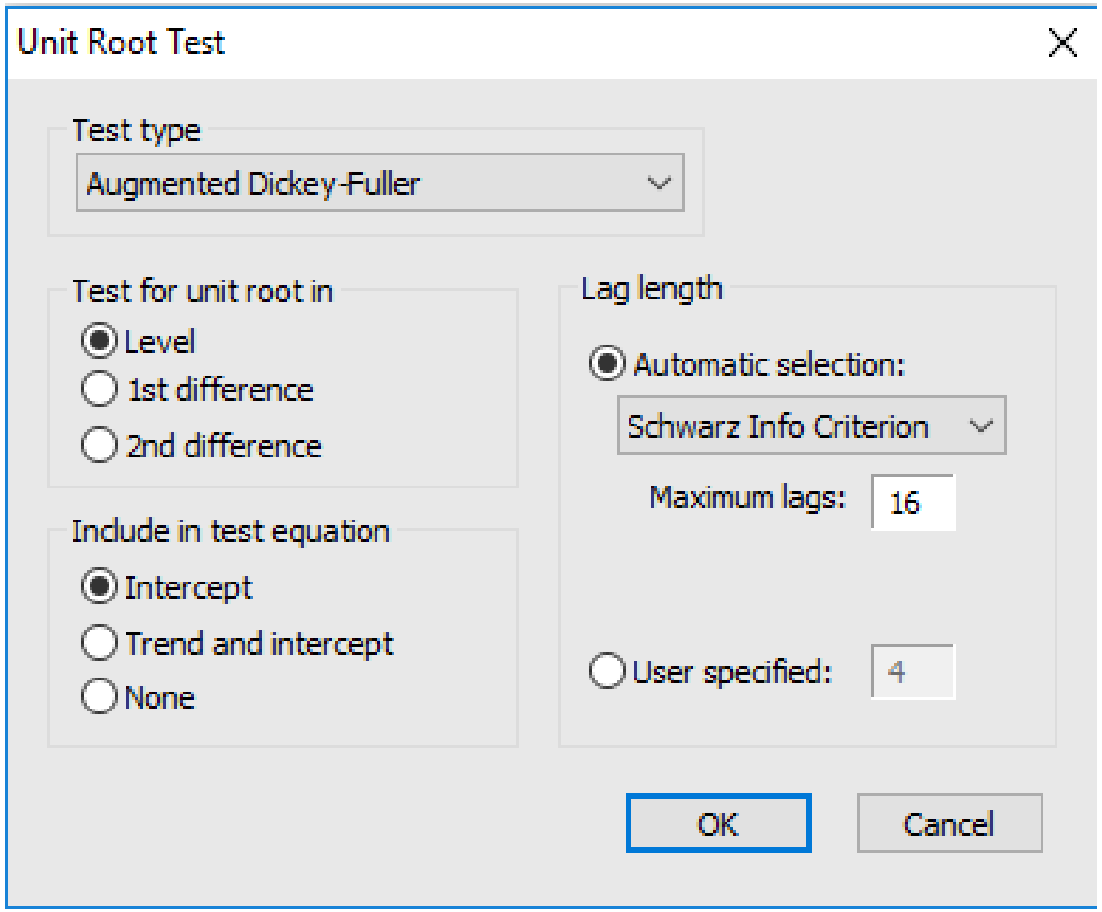
Variable-View-Graph-Line



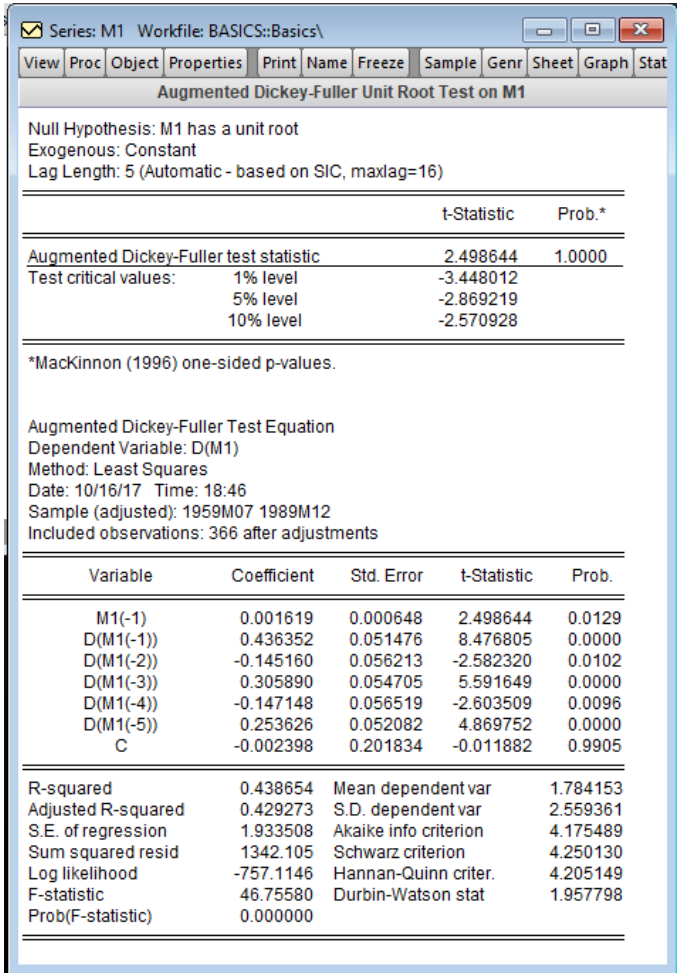
Variable-View-Unit Root Test - 1



Variable-View-Unit Root Test - 2



Variable-View-Unit Root Test - 3



Series: M1 Workfile: BASICS::Basics\

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Augmented Dickey-Fuller Unit Root Test on M1

Null Hypothesis: M1 has a unit root
Exogenous: Constant
Lag Length: 5 (Automatic - based on SIC, maxlag=16)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		2.498644	1.0000
Test critical values:	1% level	-3.448012	
	5% level	-2.869219	
	10% level	-2.570928	

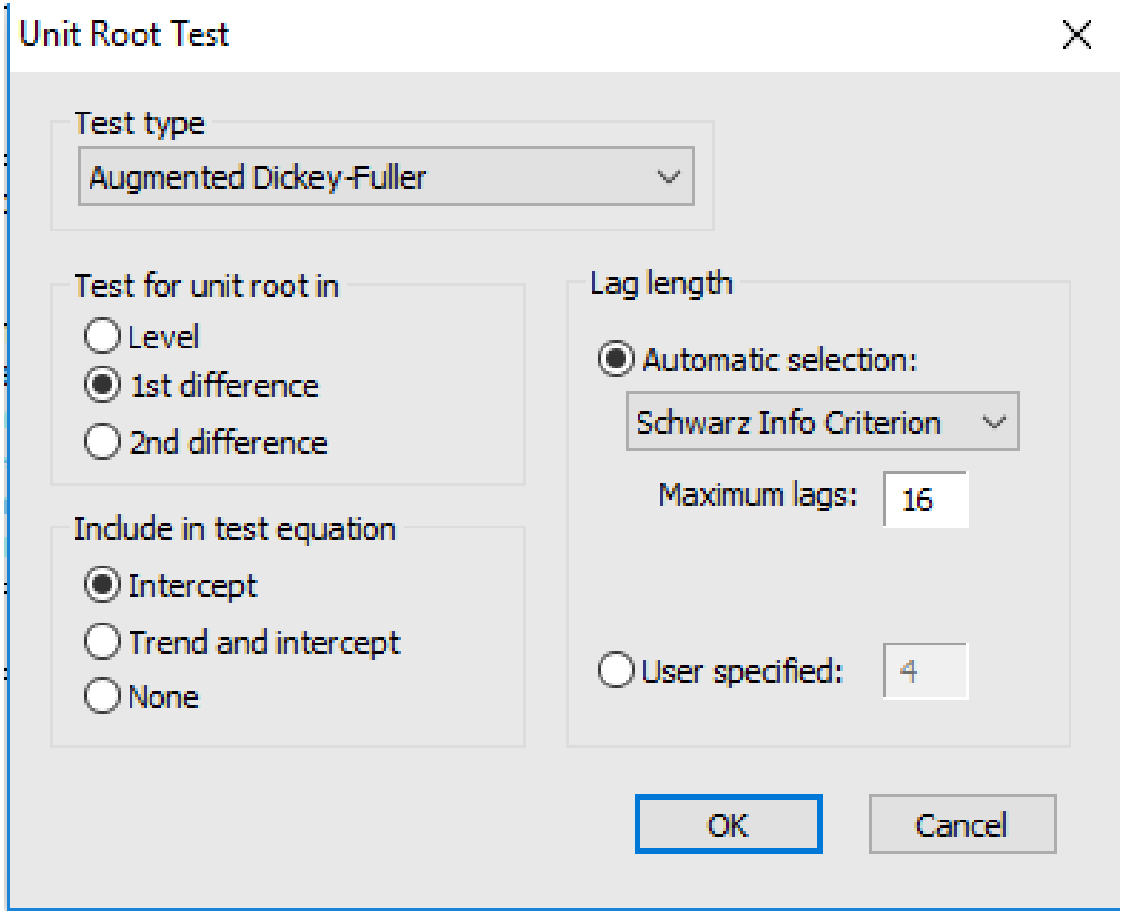
*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(M1)
Method: Least Squares
Date: 10/16/17 Time: 18:46
Sample (adjusted): 1959M07 1989M12
Included observations: 366 after adjustments

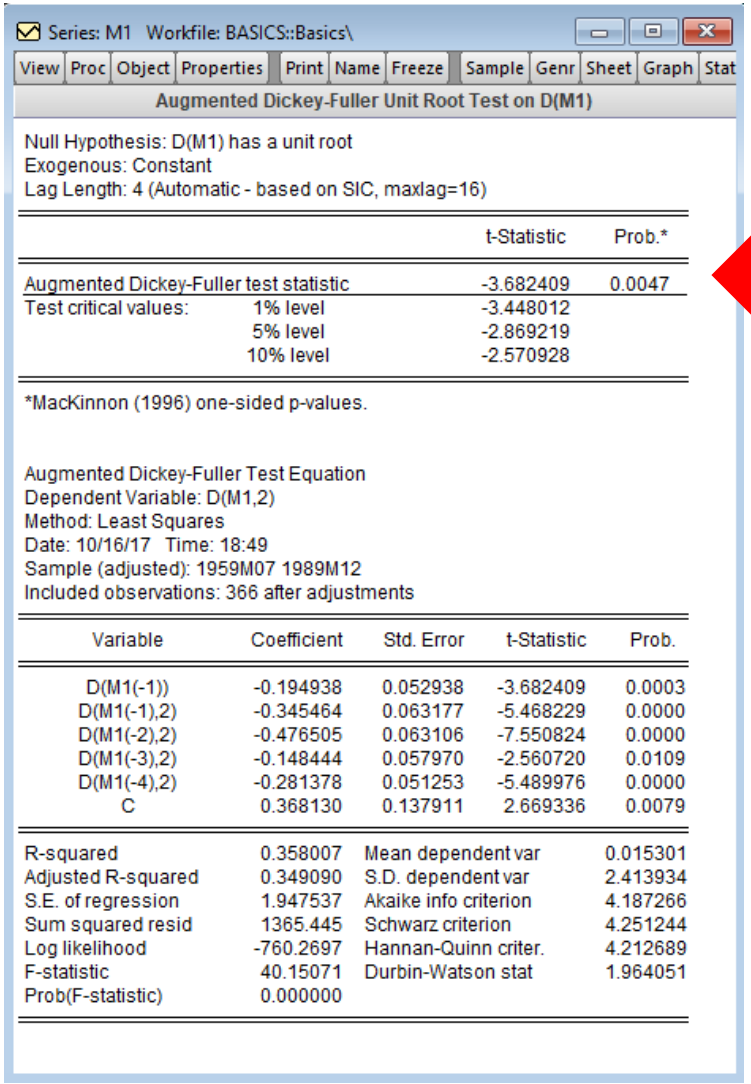
Variable	Coefficient	Std. Error	t-Statistic	Prob.
M1(-1)	0.001619	0.000648	2.498644	0.0129
D(M1(-1))	0.436352	0.051476	8.476805	0.0000
D(M1(-2))	-0.145160	0.056213	-2.582320	0.0102
D(M1(-3))	0.305890	0.054705	5.591649	0.0000
D(M1(-4))	-0.147148	0.056519	-2.603509	0.0096
D(M1(-5))	0.253626	0.052082	4.869752	0.0000
C	-0.002398	0.201834	-0.011882	0.9905

R-squared	0.438654	Mean dependent var	1.784153
Adjusted R-squared	0.429273	S.D. dependent var	2.559361
S.E. of regression	1.933508	Akaike info criterion	4.175489
Sum squared resid	1342.105	Schwarz criterion	4.250130
Log likelihood	-757.1146	Hannan-Quinn criter.	4.205149
F-statistic	46.75580	Durbin-Watson stat	1.957798
Prob(F-statistic)	0.000000		

Variable-View-Unit Root Test - 4



Variable-View-Unit Root Test - 5



Series: M1 Workfile: BASICS::Basics\

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Augmented Dickey-Fuller Unit Root Test on D(M1)

Null Hypothesis: D(M1) has a unit root
Exogenous: Constant
Lag Length: 4 (Automatic - based on SIC, maxlag=16)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.682409	0.0047
Test critical values:	1% level	-3.448012	
	5% level	-2.869219	
	10% level	-2.570928	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(M1,2)
Method: Least Squares
Date: 10/16/17 Time: 18:49
Sample (adjusted): 1959M07 1989M12
Included observations: 366 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(M1(-1))	-0.194938	0.052938	-3.682409	0.0003
D(M1(-1),2)	-0.345464	0.063177	-5.468229	0.0000
D(M1(-2),2)	-0.476505	0.063106	-7.550824	0.0000
D(M1(-3),2)	-0.148444	0.057970	-2.560720	0.0109
D(M1(-4),2)	-0.281378	0.051253	-5.489976	0.0000
C	0.368130	0.137911	2.669336	0.0079

R-squared	0.358007	Mean dependent var	0.015301
Adjusted R-squared	0.349090	S.D. dependent var	2.413934
S.E. of regression	1.947537	Akaike info criterion	4.187266
Sum squared resid	1365.445	Schwarz criterion	4.251244
Log likelihood	-760.2697	Hannan-Quinn criter.	4.212689
F-statistic	40.15071	Durbin-Watson stat	1.964051
Prob(F-statistic)	0.000000		

Model

Equation: EQ1 Workfile: BASICS::Basics\

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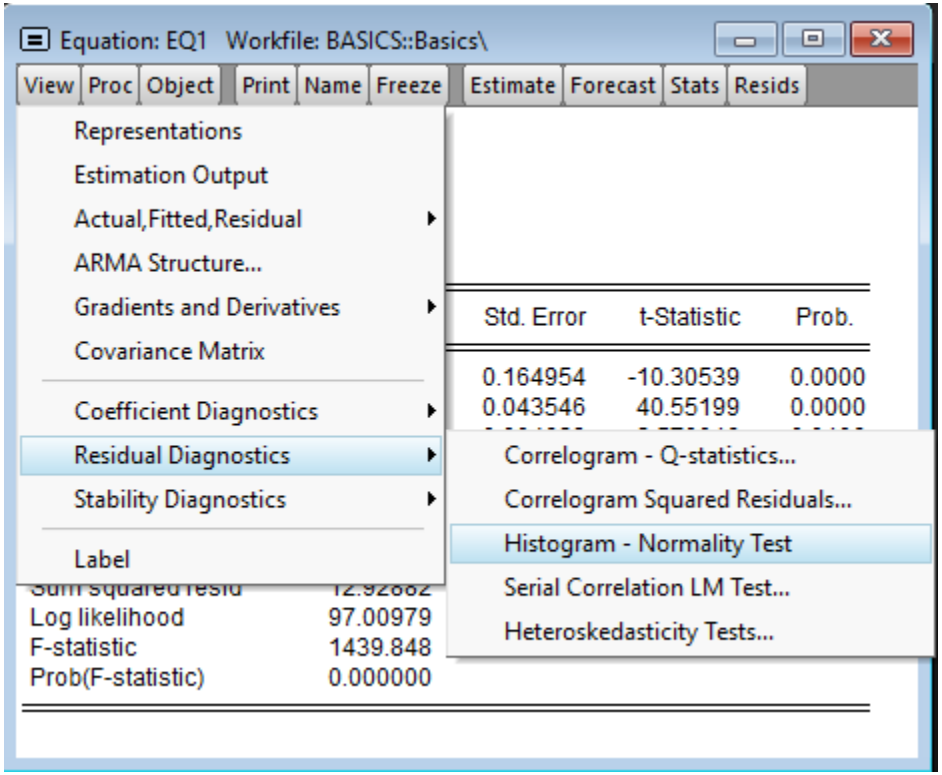
Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/16/17 Time: 18:53
Sample: 1959M01 1989M12
Included observations: 372

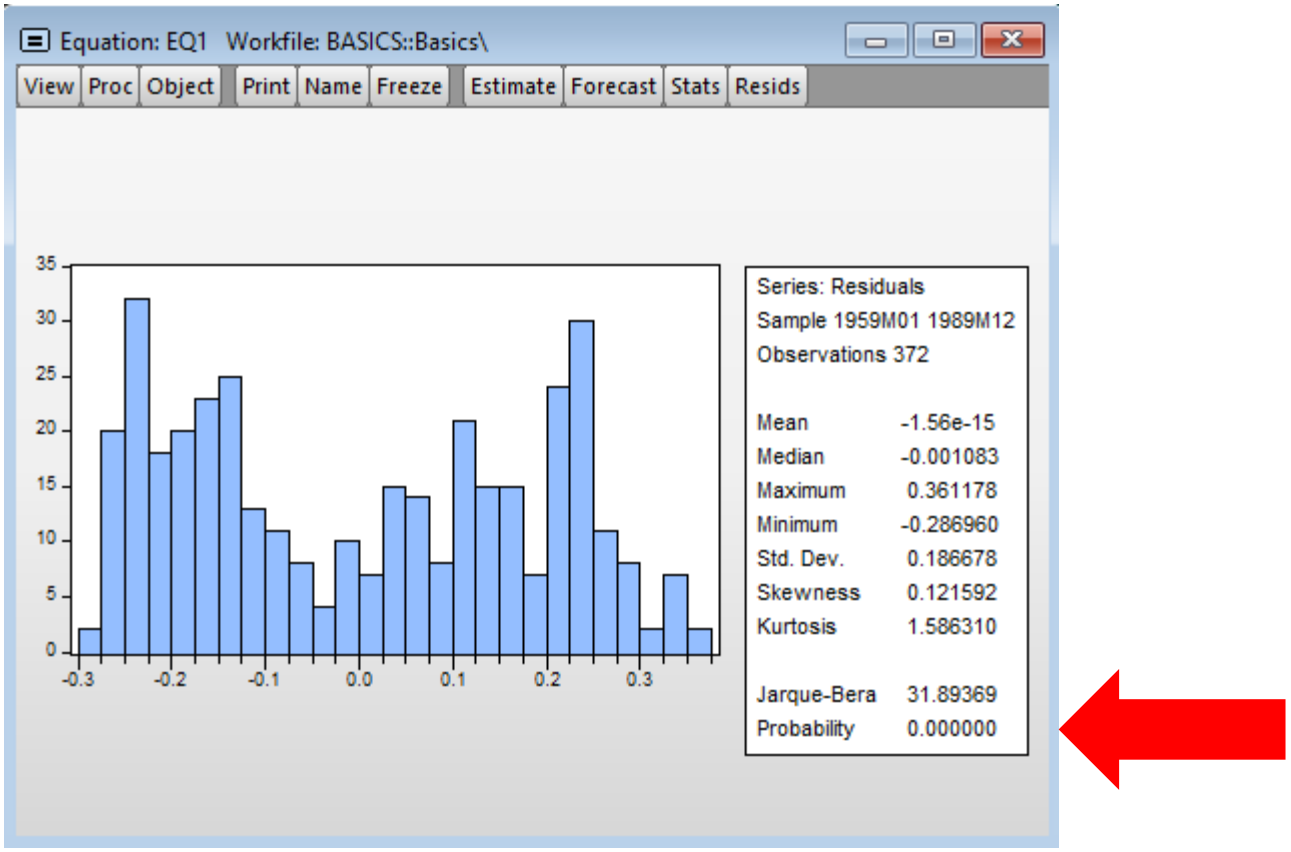
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.699912	0.164954	-10.30539	0.0000
LOG(IP)	1.765866	0.043546	40.55199	0.0000
TB3	-0.011895	0.004628	-2.570016	0.0106

R-squared	0.886416	Mean dependent var	5.663717
Adjusted R-squared	0.885800	S.D. dependent var	0.553903
S.E. of regression	0.187183	Akaike info criterion	-0.505429
Sum squared resid	12.92882	Schwarz criterion	-0.473825
Log likelihood	97.00979	Hannan-Quinn criter.	-0.492878
F-statistic	1439.848	Durbin-Watson stat	0.008687
Prob(F-statistic)	0.000000		

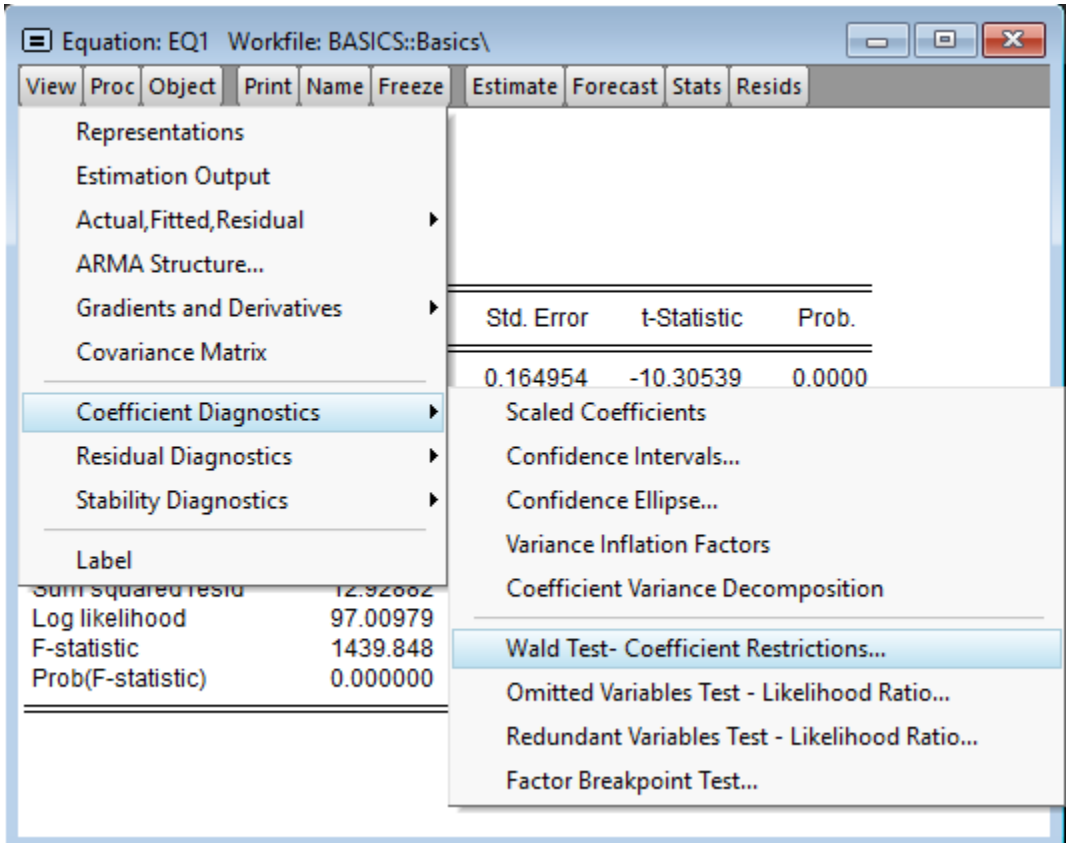
Normality test – 1



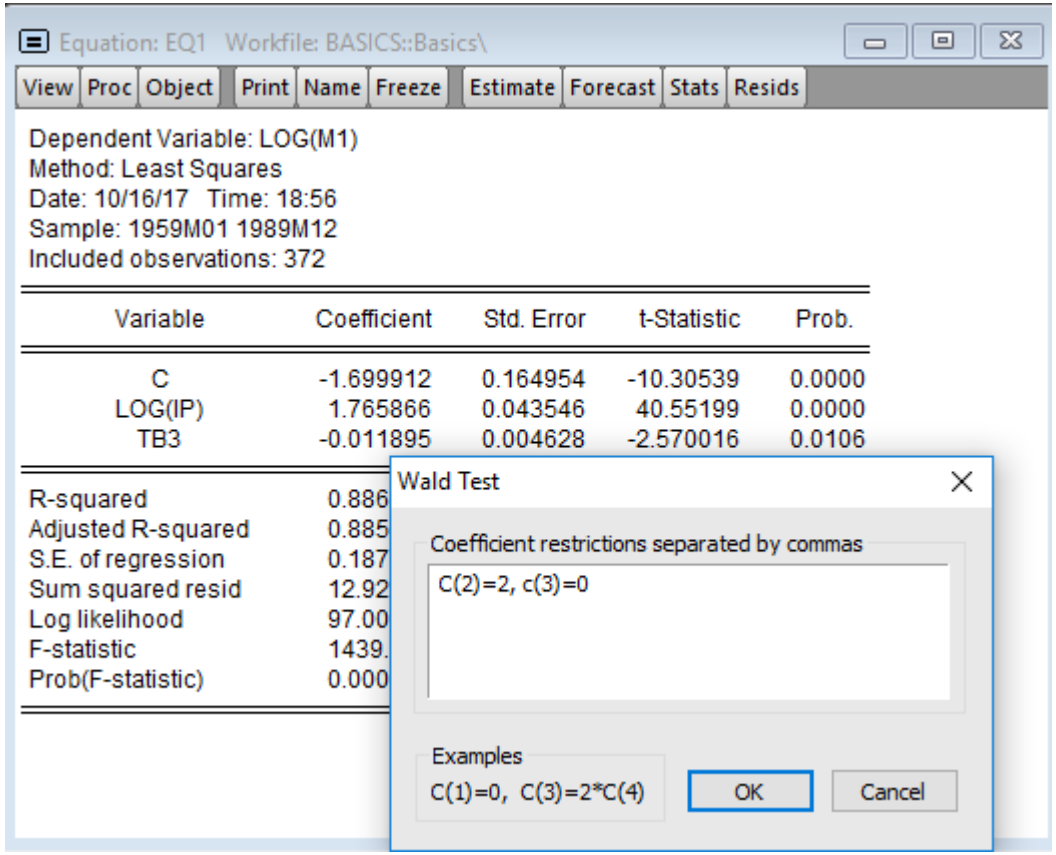
Normality test – 2



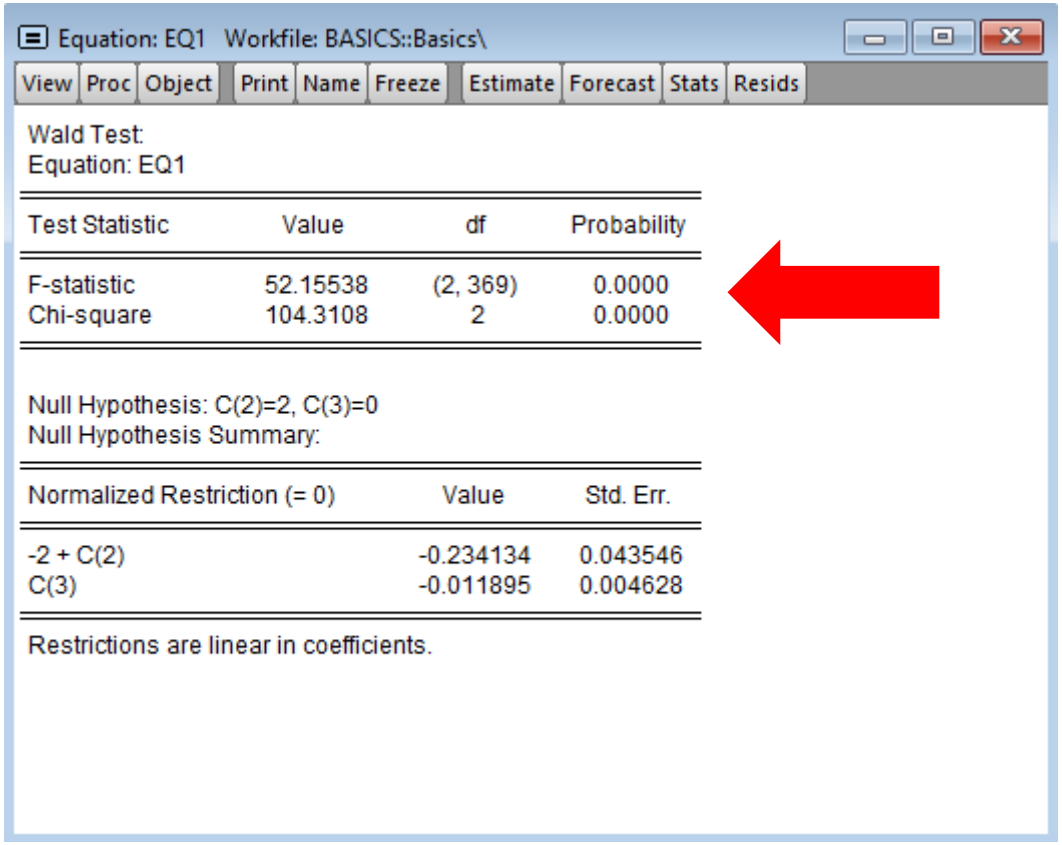
Wald test – 1


Equation: EQ1 Workfile: BASICS::Basics\
View Proc Object Print Name Freeze Estimate Forecast Stats Resids
Representations
Estimation Output
Actual,Fitted,Residual
ARMA Structure...
Gradients and Derivatives
Covariance Matrix
Coefficient Diagnostics
Residual Diagnostics
Stability Diagnostics
Label
Std. Error t-Statistic Prob.
0.164954 -10.30539 0.0000
Log likelihood 97.00979
F-statistic 1439.848
Prob(F-statistic) 0.000000
Scaled Coefficients
Confidence Intervals...
Confidence Ellipse...
Variance Inflation Factors
Coefficient Variance Decomposition
Wald Test- Coefficient Restrictions...
Omitted Variables Test - Likelihood Ratio...
Redundant Variables Test - Likelihood Ratio...
Factor Breakpoint Test...

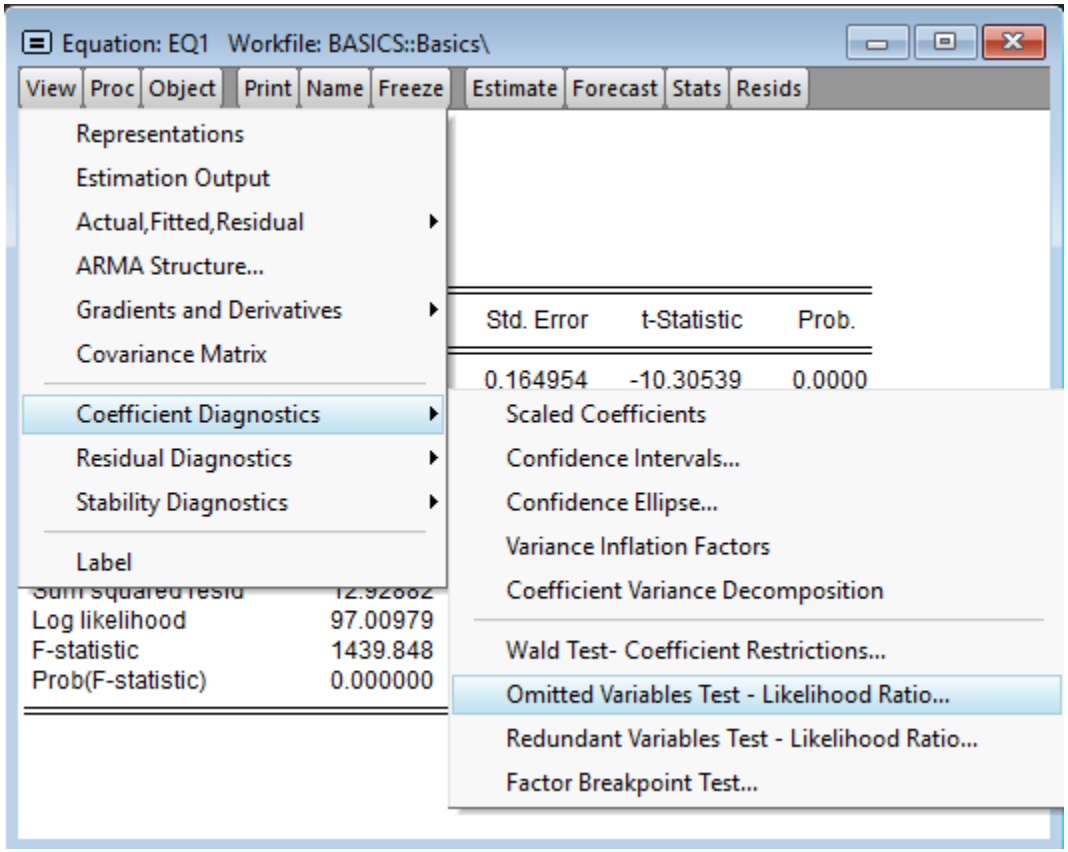
Wald test – 2


Equation: EQ1 Workfile: BASICS::Basics\
View Proc Object Print Name Freeze Estimate Forecast Stats Resids
Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/16/17 Time: 18:56
Sample: 1959M01 1989M12
Included observations: 372
Variable Coefficient Std. Error t-Statistic Prob.
C -1.699912 0.164954 -10.30539 0.0000
LOG(IP) 1.765866 0.043546 40.55199 0.0000
TB3 -0.011895 0.004628 -2.570016 0.0106
R-squared 0.886
Adjusted R-squared 0.885
S.E. of regression 0.187
Sum squared resid 12.92
Log likelihood 97.00
F-statistic 1439.
Prob(F-statistic) 0.000
Wald Test
Coefficient restrictions separated by commas
C(2)=2, c(3)=0
Examples
C(1)=0, C(3)=2*C(4)
OK
Cancel

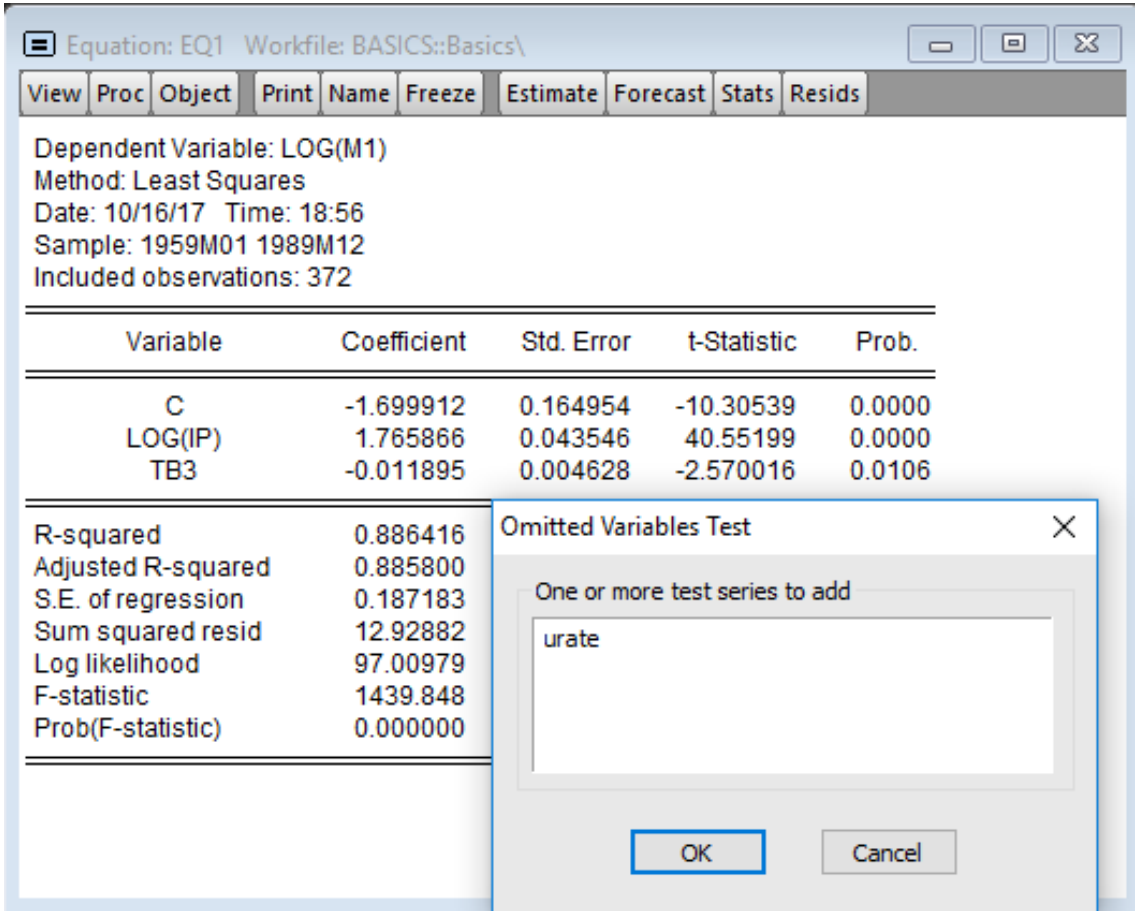
Wald test – 3


Equation: EQ1 Workfile: BASICS::Basics\
View Proc Object Print Name Freeze Estimate Forecast Stats Resids
Wald Test:
Equation: EQ1
Test Statistic Value df Probability
F-statistic 52.15538 (2, 369) 0.0000
Chi-square 104.3108 2 0.0000
Null Hypothesis: C(2)=2, C(3)=0
Null Hypothesis Summary:
Normalized Restriction (= 0) Value Std. Err.
-2 + C(2) -0.234134 0.043546
C(3) -0.011895 0.004628
Restrictions are linear in coefficients.

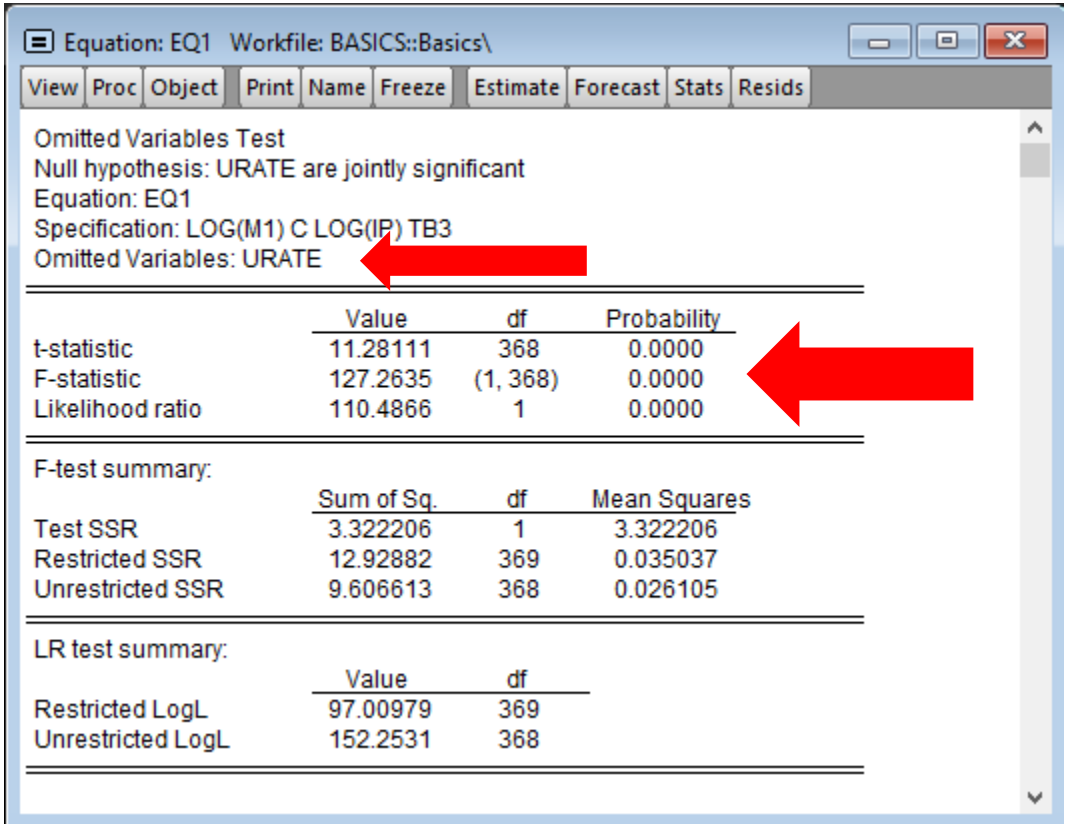
Omitted Variable Test – 1



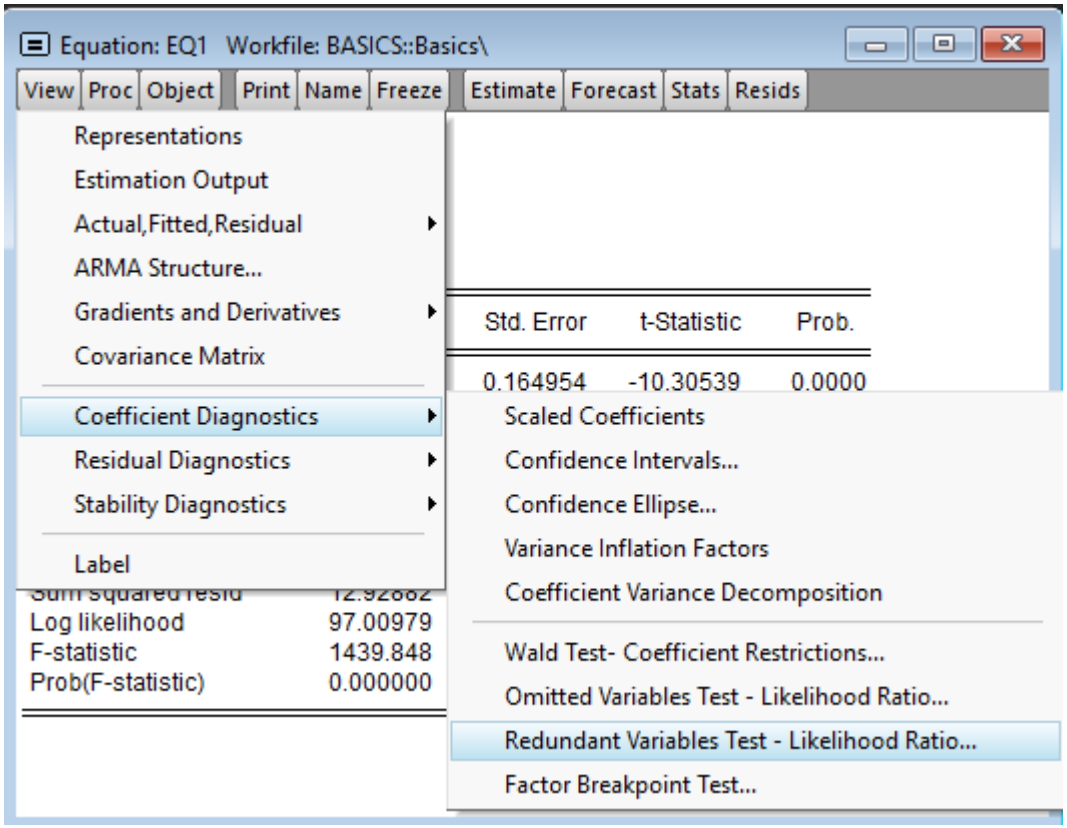
Omitted Variable Test – 2



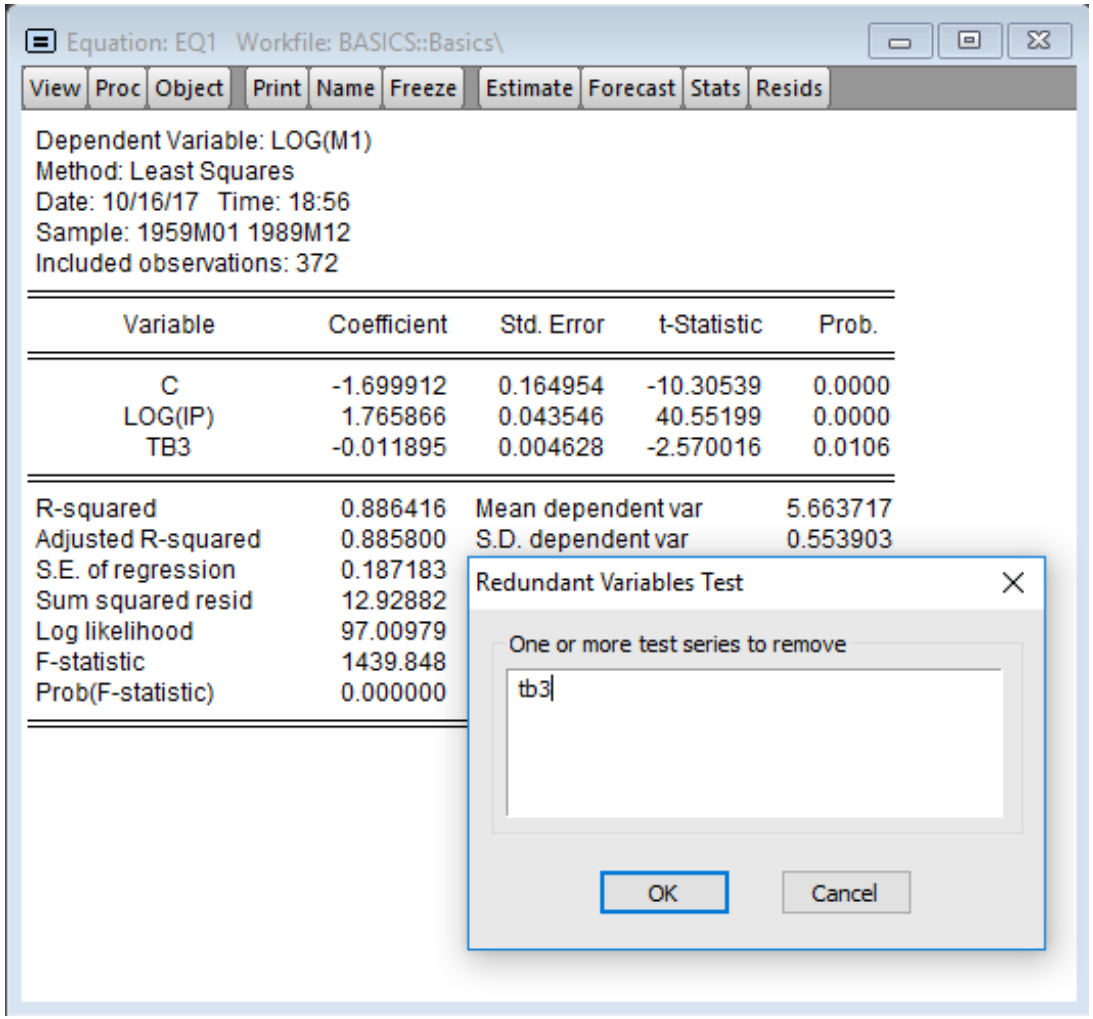
Omitted Variable Test – 3



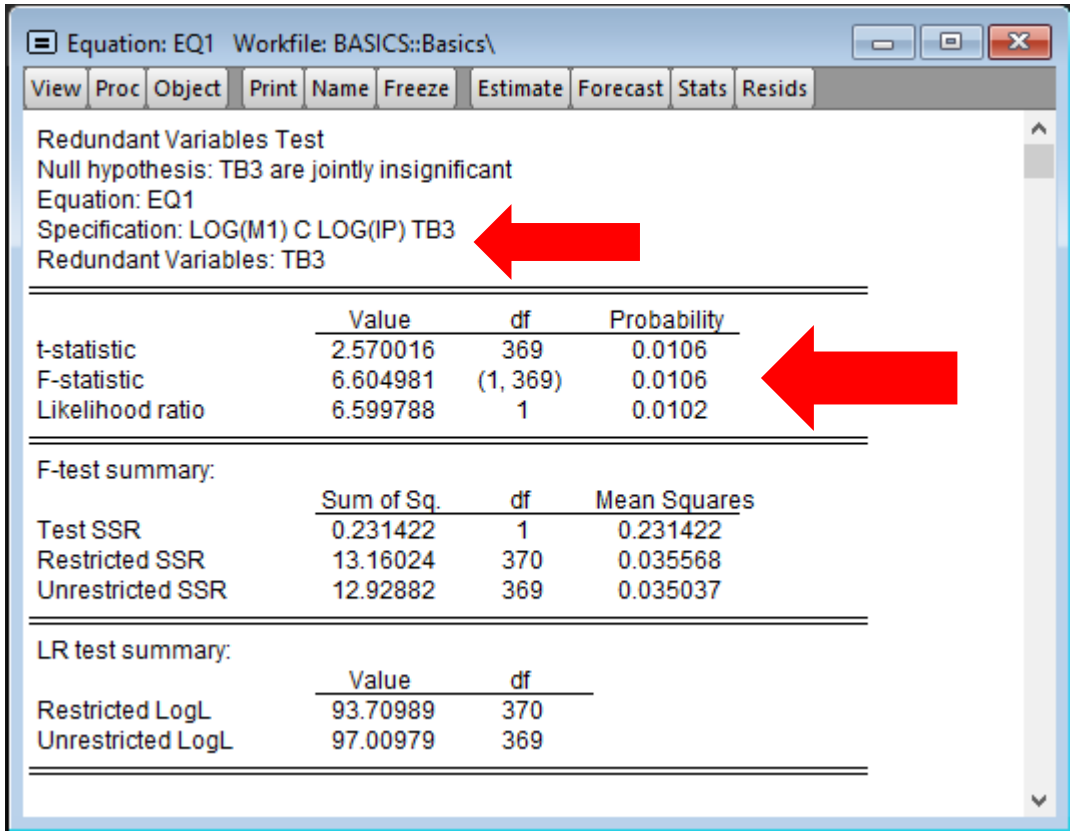
Redundant Variable Test – 1



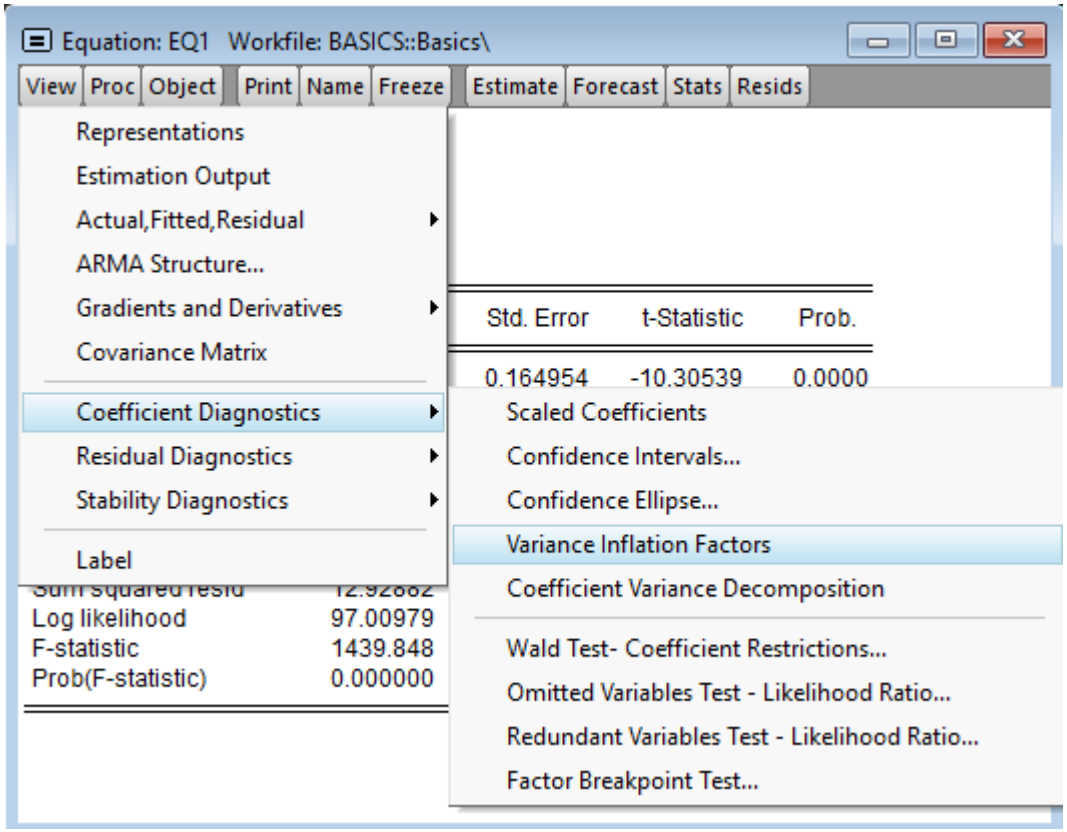
Redundant Variable Test – 2



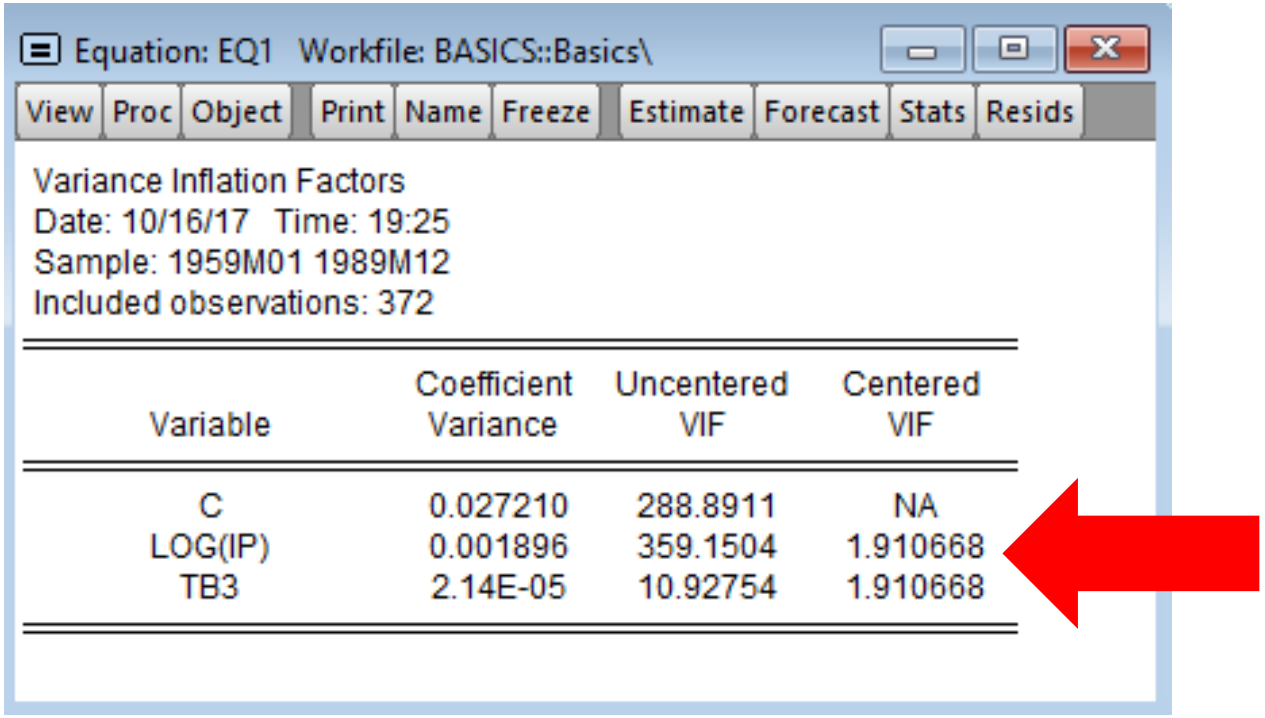
Redundant Variable Test – 3



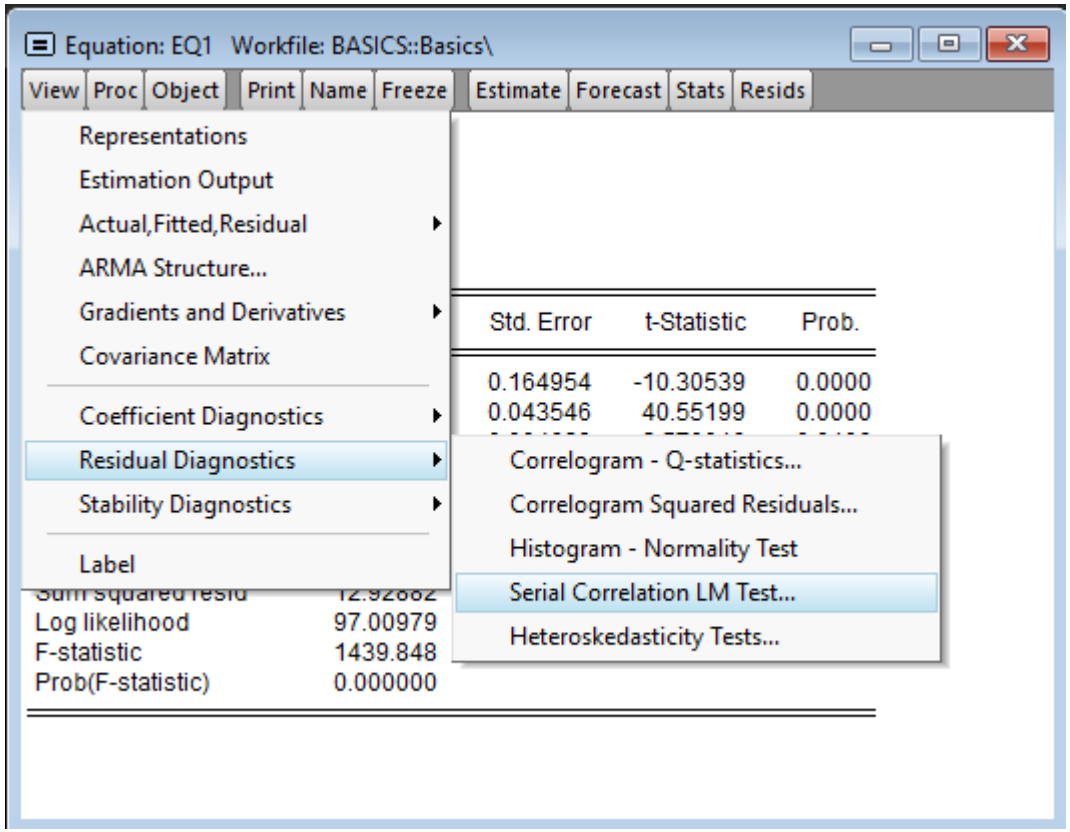
Multicolinearity test – 1



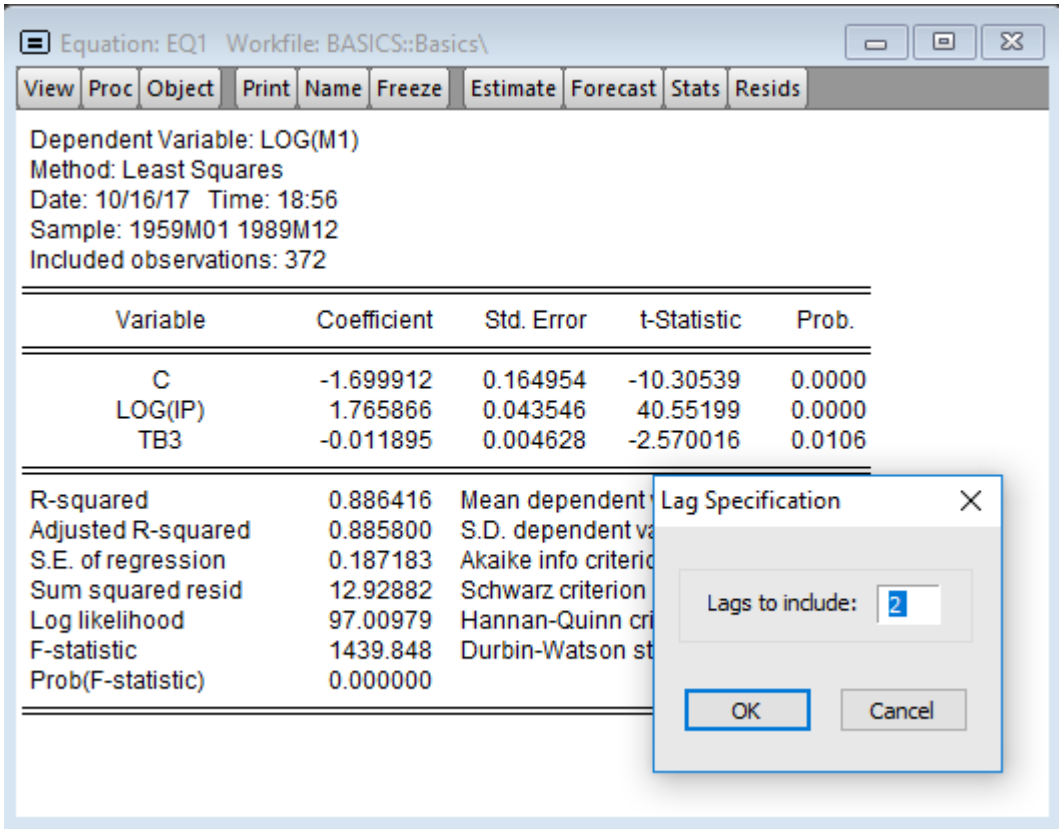
Multicolinearity test – 2



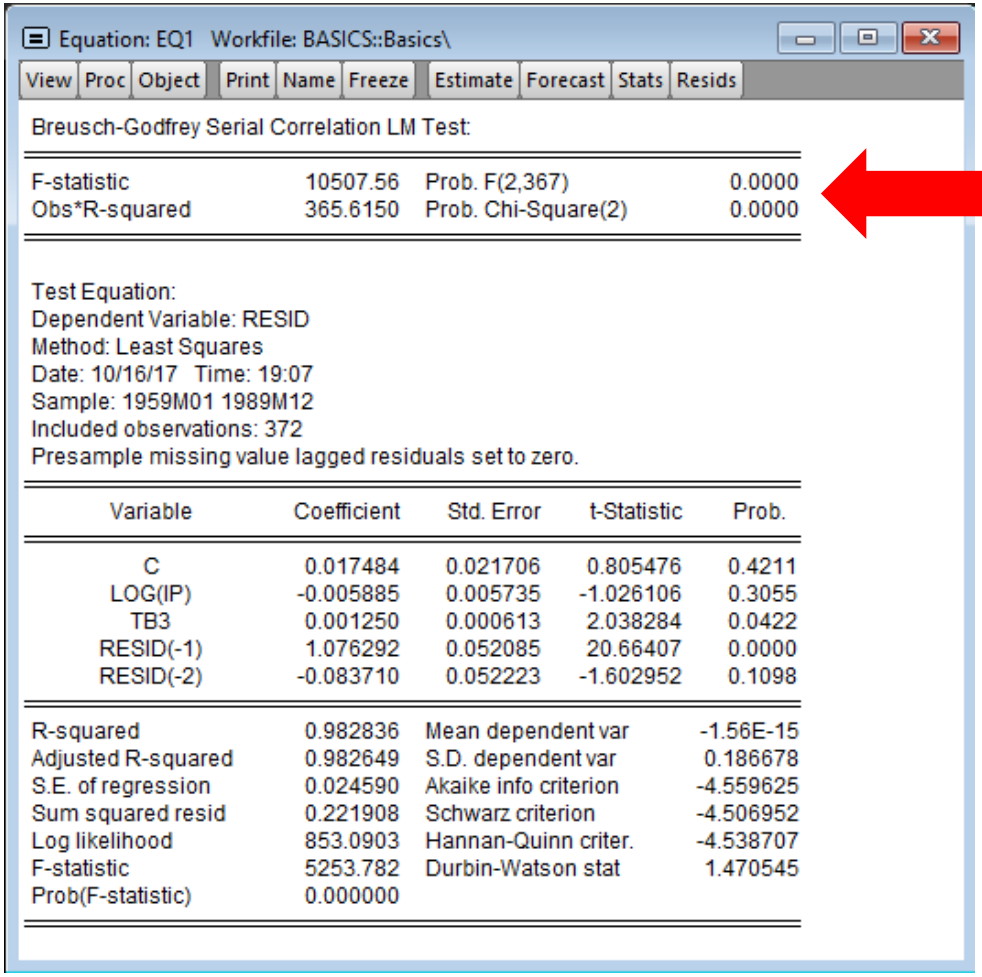
Autocorrelation test – 1



Autocorrelation test – 2



Autocorrelation test – 3



Equation: EQ1 Workfile: BASICS::Basics\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	10507.56	Prob. F(2,367)	0.0000
Obs*R-squared	365.6150	Prob. Chi-Square(2)	0.0000

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 10/16/17 Time: 19:07
Sample: 1959M01 1989M12
Included observations: 372
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.017484	0.021706	0.805476	0.4211
LOG(IP)	-0.005885	0.005735	-1.026106	0.3055
TB3	0.001250	0.000613	2.038284	0.0422
RESID(-1)	1.076292	0.052085	20.66407	0.0000
RESID(-2)	-0.083710	0.052223	-1.602952	0.1098

R-squared	0.982836	Mean dependent var	-1.56E-15
Adjusted R-squared	0.982649	S.D. dependent var	0.186678
S.E. of regression	0.024590	Akaike info criterion	-4.559625
Sum squared resid	0.221908	Schwarz criterion	-4.506952
Log likelihood	853.0903	Hannan-Quinn criter.	-4.538707
F-statistic	5253.782	Durbin-Watson stat	1.470545
Prob(F-statistic)	0.000000		

Autocorrelation test – 4

Equation: EQ1 Workfile: BASICS::Basics\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: LOG(M1)
Method: Least Squares
Date: 10/16/17 Time: 18:53
Sample: 1959M01 1989M12
Included observations: 372

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.699912	0.164954	-10.30539	0.0000
LOG(IP)	1.765866	0.043546	40.55199	0.0000
TB3	-0.011895	0.004628	-2.570016	0.0106

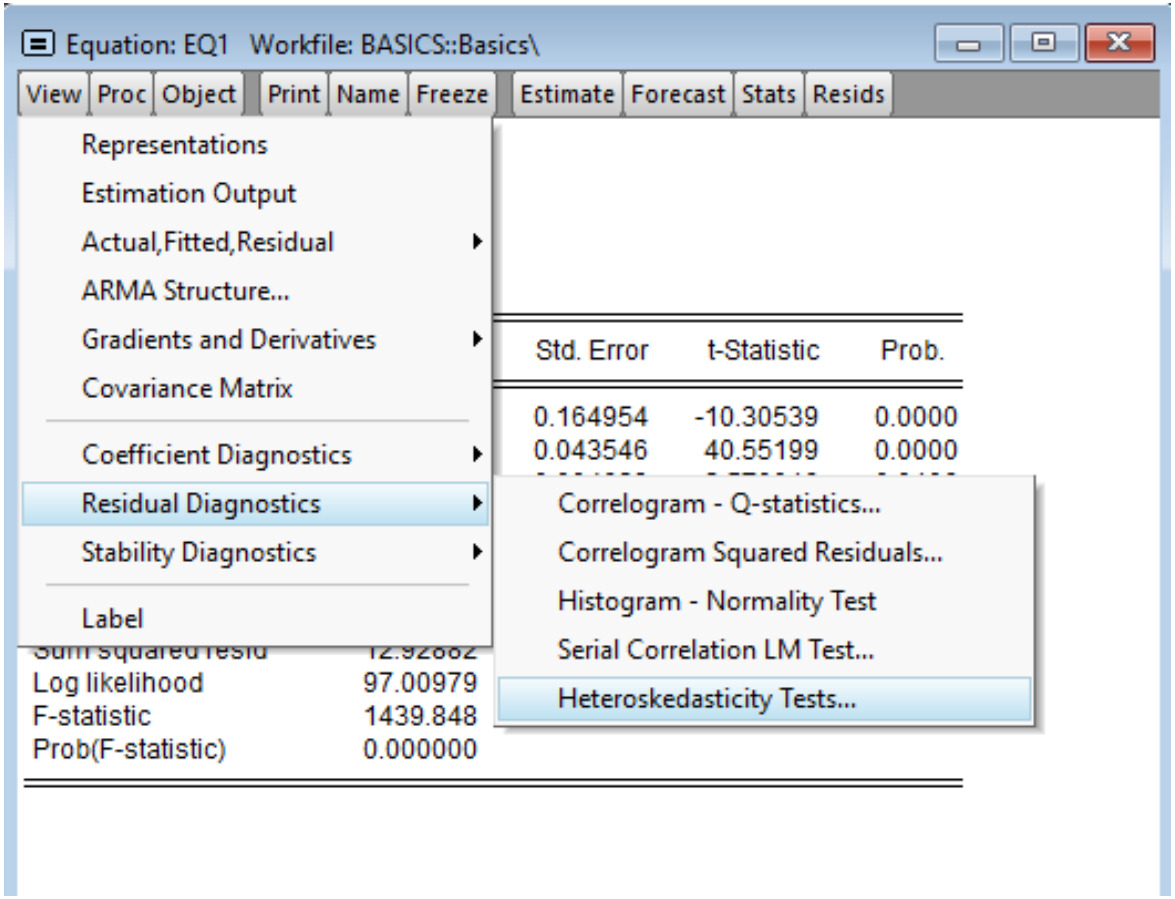
R-squared	0.886416	Mean dependent var	5.663717
Adjusted R-squared	0.885800	S.D. dependent var	0.553903
S.E. of regression	0.187183	Akaike info criterion	-0.505429
Sum squared resid	12.92882	Schwarz criterion	-0.473825
Log likelihood	97.00979	Hannan-Quinn criter.	-0.492878
F-statistic	1439.848	Durbin-Watson stat	0.008687
Prob(F-statistic)	0.000000		

Durbin–Watson test statistic d: 5% significance points of d_L and d_U.

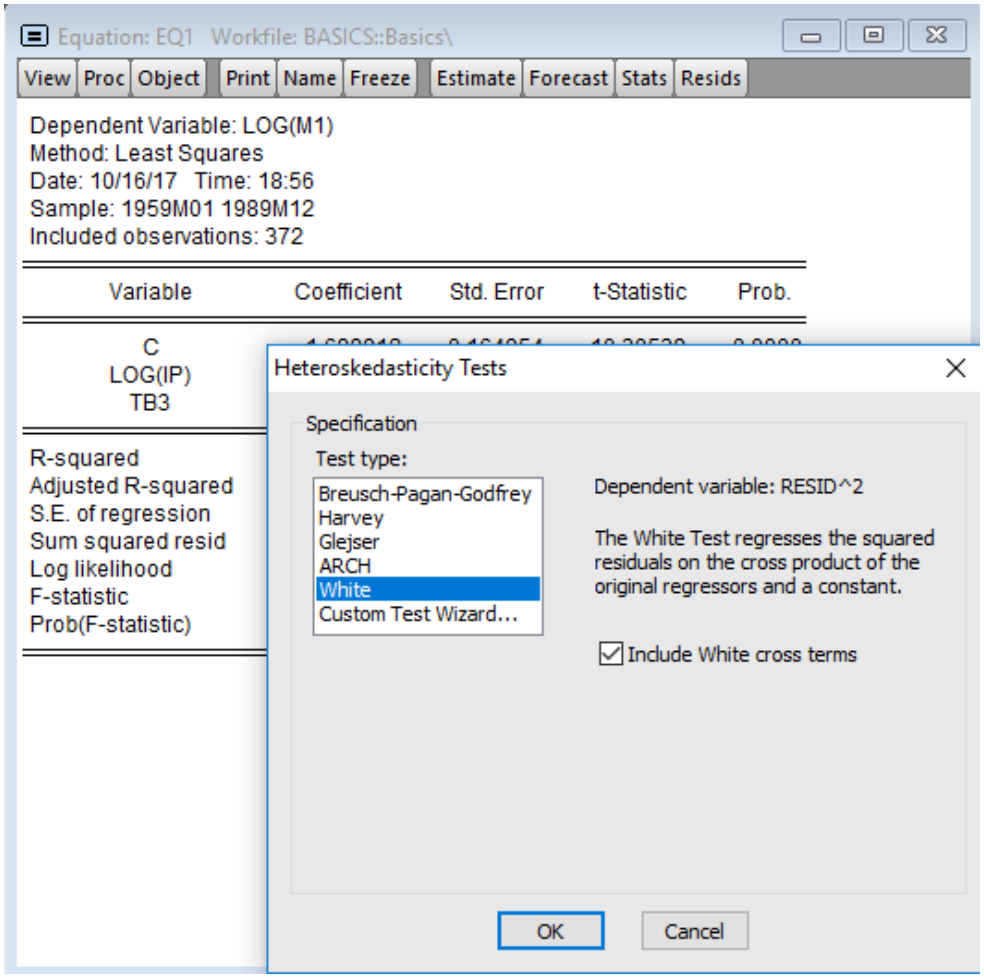
n	$k'=1$		$k'=2$		$k'=3$		$k'=4$		$k'=5$	
	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
15	1.08	1.36	0.95	1.54	0.82	1.75	0.69	1.97	0.56	2.21
16	1.10	1.37	0.98	1.54	0.86	1.73	0.74	1.93	0.62	2.15
17	1.13	1.38	1.02	1.54	0.90	1.71	0.78	1.90	0.67	2.10
18	1.16	1.39	1.05	1.53	0.93	1.69	0.82	1.87	0.71	2.06
19	1.18	1.40	1.08	1.53	0.97	1.68	0.86	1.85	0.75	2.02
20	1.20	1.41	1.10	1.54	1.00	1.68	0.90	1.83	0.79	1.99
21	1.22	1.42	1.13	1.54	1.03	1.67	0.93	1.81	0.83	1.96
22	1.24	1.43	1.15	1.54	1.05	1.66	0.96	1.80	0.86	1.94
23	1.26	1.44	1.17	1.54	1.08	1.66	0.99	1.79	0.90	1.92
24	1.27	1.45	1.19	1.55	1.10	1.66	1.01	1.78	0.93	1.90
25	1.29	1.45	1.21	1.55	1.12	1.66	1.04	1.77	0.95	1.89
26	1.30	1.46	1.22	1.55	1.14	1.65	1.06	1.76	0.98	1.88
27	1.32	1.47	1.24	1.56	1.16	1.65	1.08	1.76	1.01	1.86
28	1.33	1.48	1.26	1.56	1.18	1.65	1.10	1.75	1.03	1.85
29	1.34	1.48	1.27	1.56	1.20	1.65	1.12	1.74	1.05	1.84
30	1.35	1.49	1.28	1.57	1.21	1.65	1.14	1.74	1.07	1.83
31	1.36	1.50	1.30	1.57	1.23	1.65	1.16	1.74	1.09	1.83
32	1.37	1.50	1.31	1.57	1.24	1.65	1.18	1.73	1.11	1.82
33	1.38	1.51	1.32	1.58	1.26	1.65	1.19	1.73	1.13	1.81
34	1.39	1.51	1.33	1.58	1.27	1.65	1.21	1.73	1.15	1.81
35	1.40	1.52	1.34	1.58	1.28	1.65	1.22	1.73	1.16	1.80
36	1.41	1.52	1.35	1.59	1.29	1.65	1.24	1.73	1.18	1.80
37	1.42	1.53	1.36	1.59	1.31	1.66	1.25	1.72	1.19	1.80
38	1.43	1.54	1.37	1.59	1.32	1.66	1.26	1.72	1.21	1.79
39	1.43	1.54	1.38	1.60	1.33	1.66	1.27	1.72	1.22	1.79
40	1.44	1.54	1.39	1.60	1.34	1.66	1.29	1.72	1.23	1.79
45	1.48	1.57	1.43	1.62	1.38	1.67	1.34	1.72	1.29	1.78
50	1.50	1.59	1.46	1.63	1.42	1.67	1.38	1.72	1.34	1.77
55	1.53	1.60	1.49	1.64	1.45	1.68	1.41	1.72	1.38	1.77
60	1.55	1.62	1.51	1.65	1.48	1.69	1.44	1.73	1.41	1.77
65	1.57	1.63	1.54	1.66	1.50	1.70	1.47	1.73	1.44	1.77
70	1.58	1.64	1.55	1.67	1.52	1.70	1.49	1.74	1.46	1.77
75	1.60	1.65	1.57	1.68	1.54	1.71	1.51	1.74	1.49	1.77
80	1.61	1.66	1.59	1.69	1.56	1.72	1.53	1.74	1.51	1.77
85	1.62	1.67	1.60	1.70	1.57	1.72	1.55	1.75	1.52	1.77
90	1.63	1.68	1.61	1.70	1.59	1.73	1.57	1.75	1.54	1.78
95	1.64	1.69	1.62	1.71	1.60	1.73	1.58	1.75	1.56	1.78
100	1.65	1.69	1.63	1.72	1.61	1.74	1.59	1.76	1.57	1.78

n = number of observations
k' = number of explanatory variables

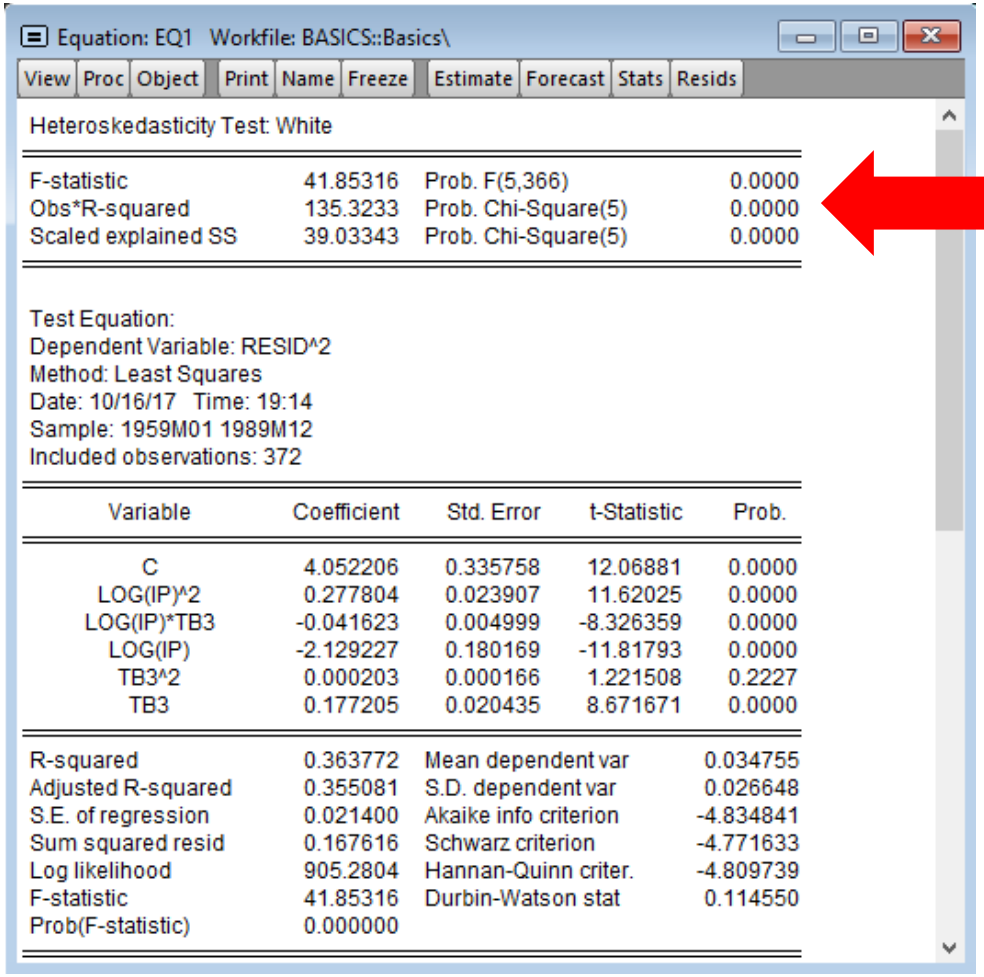
Heteroskedasticity Tests – 1



Heteroskedasticity Tests – 2



Heteroskedasticity Tests – 3



Equation: EQ1 Workfile: BASICS::Basics\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Heteroskedasticity Test: White

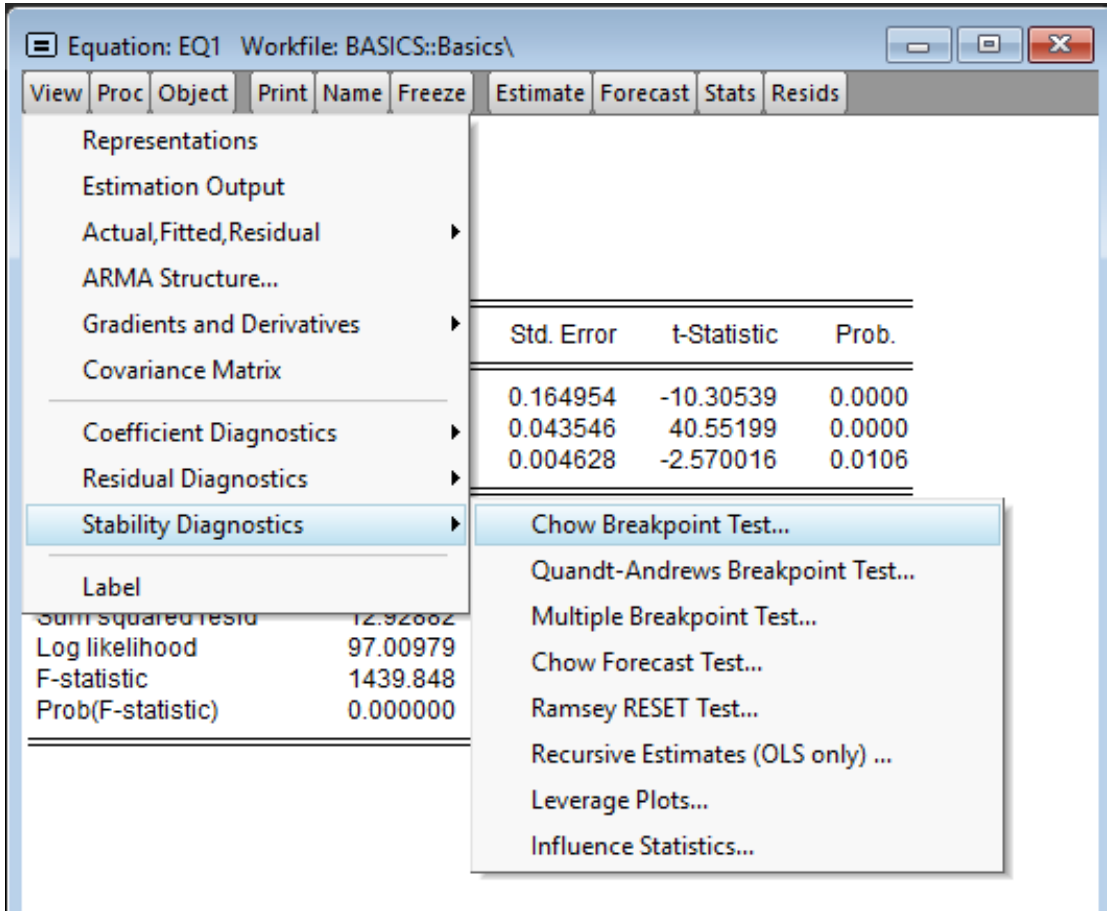
F-statistic	41.85316	Prob. F(5,366)	0.0000
Obs*R-squared	135.3233	Prob. Chi-Square(5)	0.0000
Scaled explained SS	39.03343	Prob. Chi-Square(5)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 10/16/17 Time: 19:14
Sample: 1959M01 1989M12
Included observations: 372

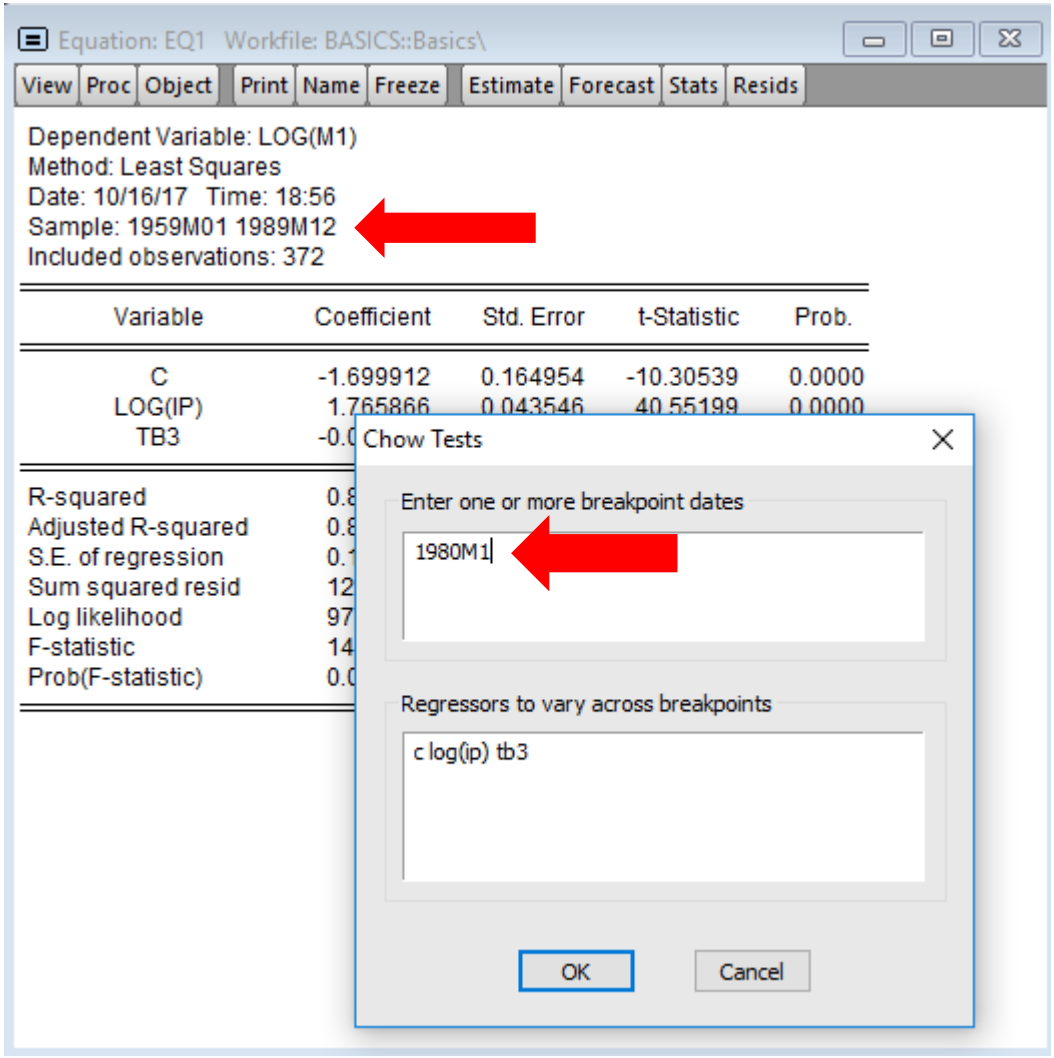
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.052206	0.335758	12.06881	0.0000
LOG(IP)^2	0.277804	0.023907	11.62025	0.0000
LOG(IP)*TB3	-0.041623	0.004999	-8.326359	0.0000
LOG(IP)	-2.129227	0.180169	-11.81793	0.0000
TB3^2	0.000203	0.000166	1.221508	0.2227
TB3	0.177205	0.020435	8.671671	0.0000

R-squared	0.363772	Mean dependent var	0.034755
Adjusted R-squared	0.355081	S.D. dependent var	0.026648
S.E. of regression	0.021400	Akaike info criterion	-4.834841
Sum squared resid	0.167616	Schwarz criterion	-4.771633
Log likelihood	905.2804	Hannan-Quinn criter.	-4.809739
F-statistic	41.85316	Durbin-Watson stat	0.114550
Prob(F-statistic)	0.000000		

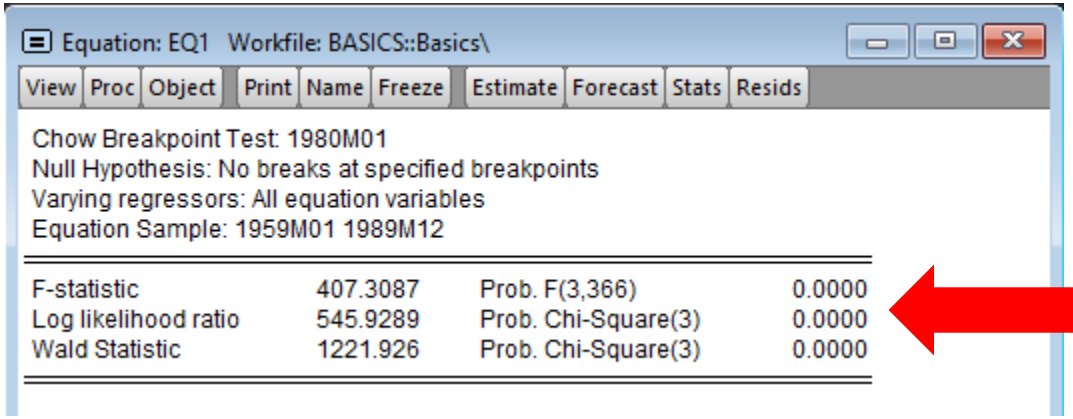
Stability test – 1



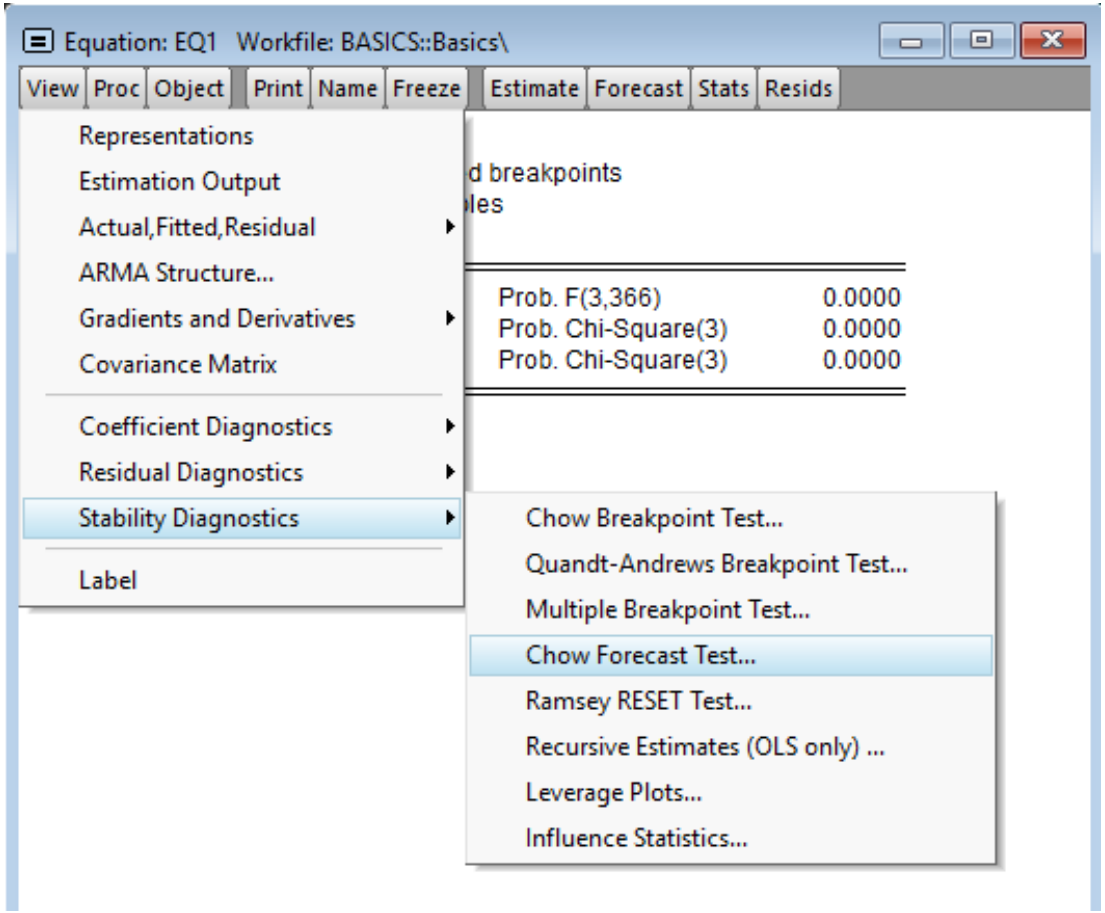
Stability test – 2



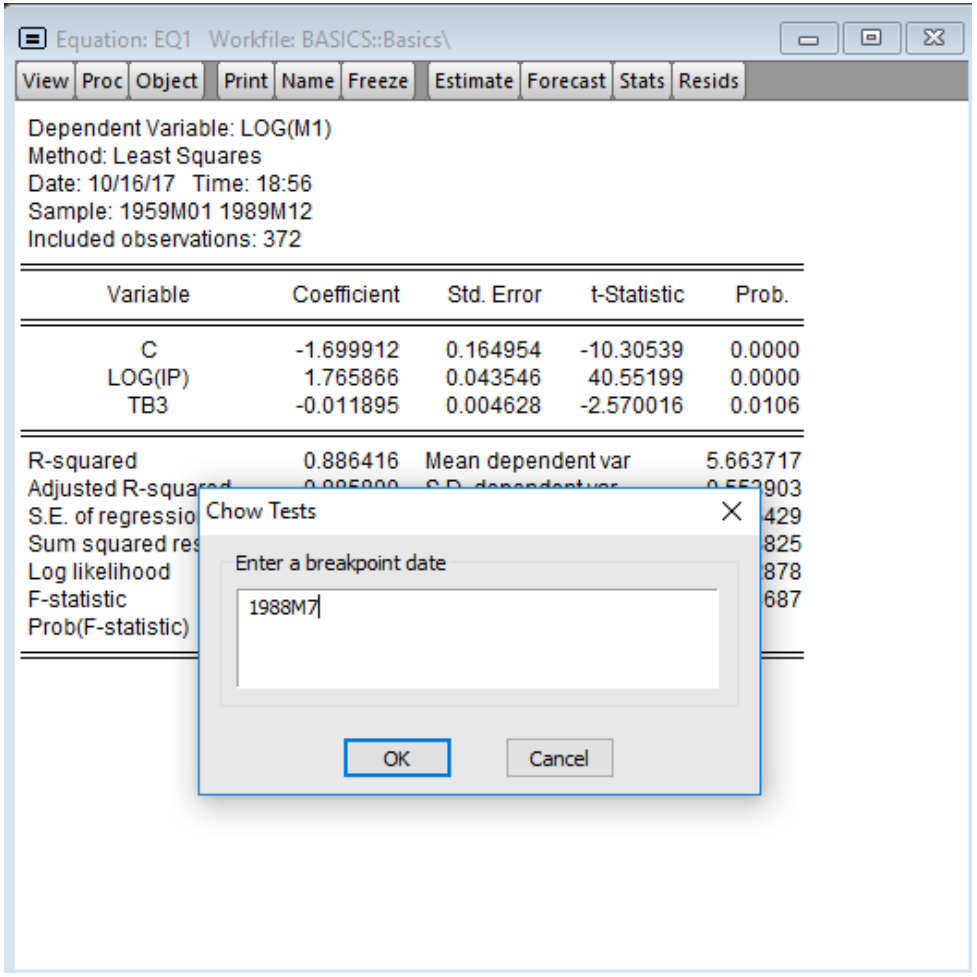
Stability test – 3



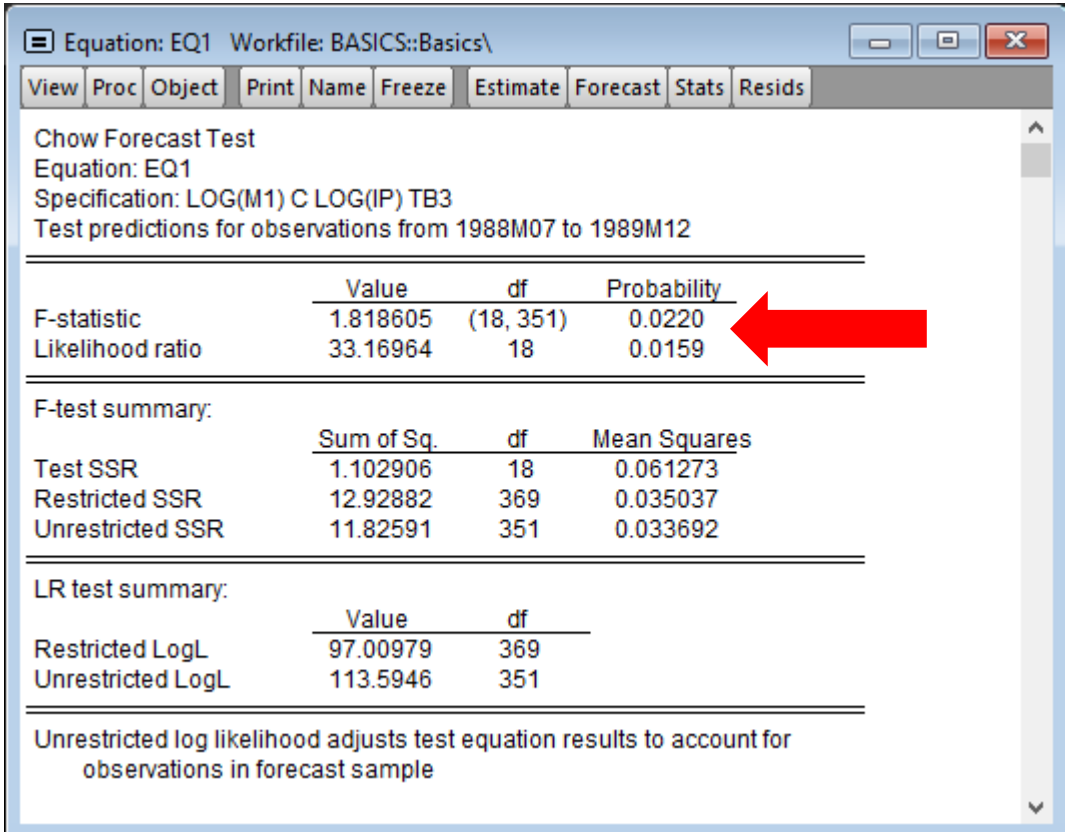
Stability test – 4



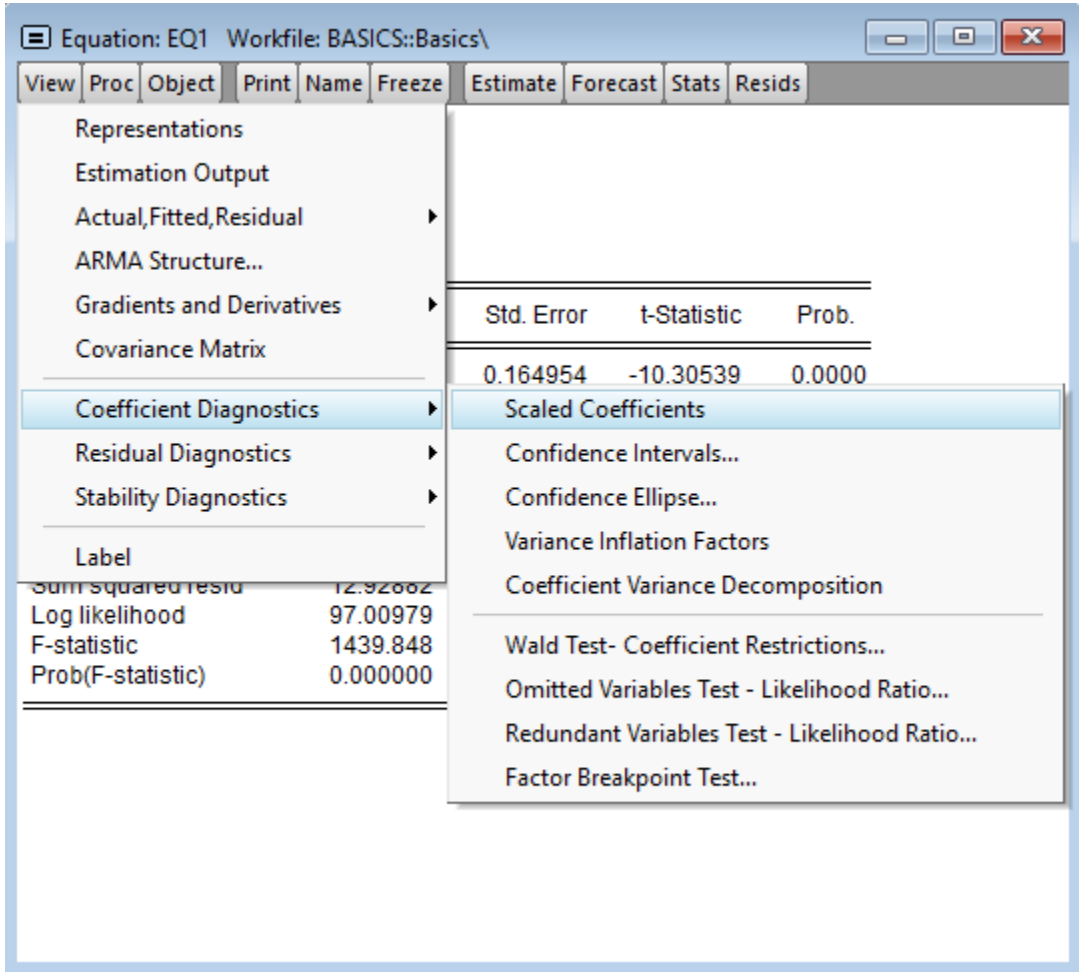
Stability test – 5



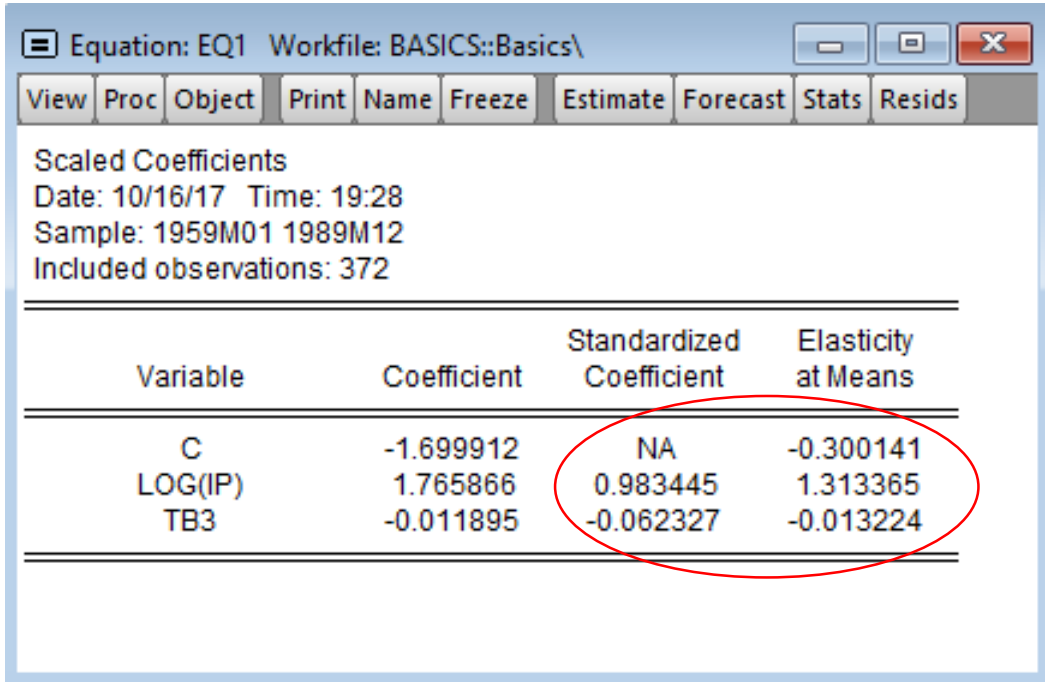
Stability test – 6



Elasticity and normalized coefficients – 1



Elasticity and normalized coefficients – 2



Review

KSE

Linear regression

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \ldots + \beta_{k-1} x_{k-1t} + \varepsilon_t, t = \overline{1,n}$$

y_t - dependent variable;

$x_{1t}, x_{2t}, \ldots, x_{k-1t}$ - independent variables;

ε_t - residuals.

Multiple Regression Tests

- Test residual for normality
- Test parameter significance
  - Overall model
  - Individual coefficients
- Test for multicollinearity
- Test for model stability
- Test for residuals autocorrelation
- Test for residuals homoscedasticity
- Test for specification
- Test for stationary process

Thank you for attention!

KSE